**The Prehistoric Origins of Mathematics** »

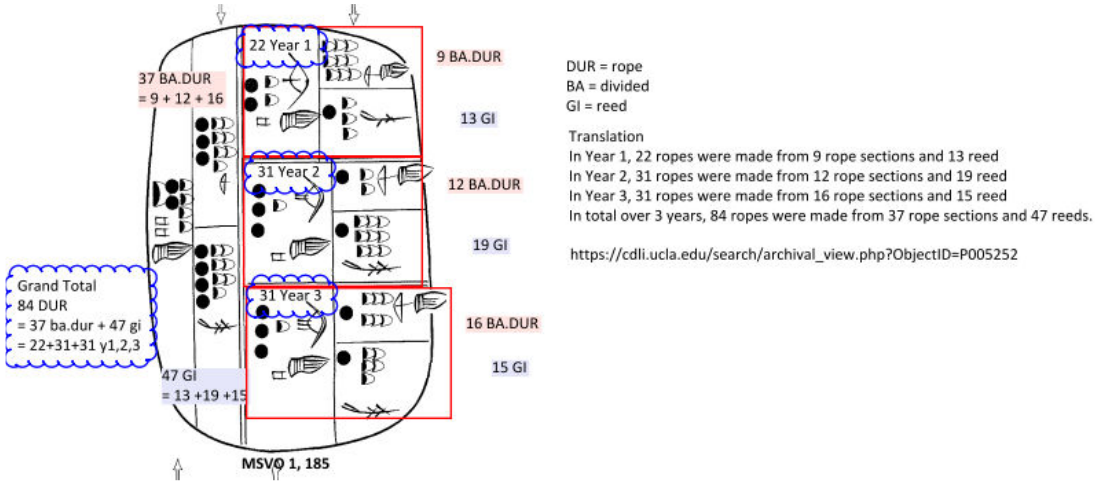
# The Mathematics of Uruk and Susa (c.3500-3000 BCE)

By **Assad Ebrahim**, on December 28th, 2019

*Topic:* ***General Interest***, ***History***, ***Mathematics***

*Part 2 in History of Mathematics series. Part 1 is* ***The Prehistoric Origins of Mathematics****.*

**Summary** The written mathematics of late neolithic Mesopotamians emerged from a cultural impetus to control the flow of surplus economic goods in their settled societies: grains and grain products, sheep and other herded animals, jugs of dairy fats and beer, rope, textiles, etc. It then progressed, as all human inventions do, through a graded sequence of innovations, each expanding upon and rendering more efficient the ability to communicate and record quantitative information. Success relied on the use of visual signs whose meaning was accepted by convention, and whose widening repertoire was standardized through scribal schools associated with the temple economy of the early city-state (initially Uruk c.3500 BCE). **Mathematically**, quantity was governed by a comprehensive set of metrological systems each with its own factor relations, sums were made which required grouping and replacement operations to ensure all quantities were written in canonical form, and fractions and scaling were used as a matter of course to document production inputs. By the end of the fourth millennium BCE, economic control through writing and mathematics was a standard part of how city-states were run, touching off a 1000-year period in which Sumerian city-states would joust for dominance (Early Dynastic period).




Tablet MSVO 1,185 during Uruk III period (3200-3000 BCE) shows detailed accounting of the production of ropes (DUR) from inputs pre-existing pieces (BA.DUR) and new reed (GI). Source: Englund/1998, p.63

Economically, the relevant innovations are (1) administrative seals facilitating economic control from 6000 BCE in the Ubaid period, (2) clay accounting tokens in use between 6000 and 3500 BCE to keep the count, (3) clay envelopes for storing collections of tokens that arose from 3500 BCE as an innovation to allow storing a transaction, (4) numerical tablets recording quantity, initially on the surface of the clay envelopes, then afterward on their own tablets, independent of token counters, (5) numero-ideographic tablets allowing documentation of quantity as well as explicitly commodity which her previously been implicitly known from context, and (6) proto-cuneiform detailed text accounts containing lists and comprehensive administrative records. These developments in economic control and writing, centered around the transactional mathematics of bookkeeping, are visible in the archaeological layers in Uruk and

Susa and in other sites of comparable size in the period.

Uruk was the hegemonic centre of this innovation, starting from 3500 BCE. The increased economic control that writing and mathematics supplied during its first 500 years (from 3500-3000 BCE) generated economic efficiencies that accelerated Uruk's growth and dominance over its neighbors, radiating the new inventions outward throughout the Greater Mesopotamian region, up to Anatolia in the North. It also set in motion a hypertrophic bureaucratic administrative culture that, over the next 1000 years would culminate in the ambitious Ur III program of controlling an empire's economics through mathematics (c.2000 BCE). (We can see this increased mathematical adventurousness in Ur III in e.g. the theoretical simulation of a cattle-herd population with projected economic yields for 10 future years, solving, in modern terms, a differential equation in table form, cf. the cuneiform tablet **TCL 2, no.5499**, **Nissen/1993**, p.97-102.)

In this paper, we will look at the mathematical developments from 3500-3000 BCE that gave rise to the Uruk-template society in the Near East. This is in line with the overall thesis of the History of Mathematics series that mathematics, technology, and culture are inextricably linked. Changes in the one stimulate and catalyze changes in the other, each of which can change profoundly the subsequent trajectory of societal development.

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## 1. The urban mathematics of Uruk and Susa

As we have seen in Part One paper **Prehistoric Origins of Mathematics**, written mathematics originated with scribal record-keeping (bookkeeping) associated with the redistributive temple-economy of the largest Neolithic cities of Sumerian Mesopotamia (southern Iraq) and Elamite Khuzistan (western Iraq). Temple bookkeepers accounted for products of staple finance and surplus goods arising from irrigation agriculture supplemented by herding, fishing, and hunting. This was the time of the ascendancy of Uruk in the southern Mesopotamia, its excursions to the east (Aratta, Awan, Susa), the time before the Flood (c.2900 BCE), and before these economic and adminstrative innovations became standard through the region.

An impetus toward an accounting function was the use of communal labor to increase the productivity of the community through large-scale efforts such as irrigation canals, cultivation of broader plots of land, care of larger sized herds, the manufacture of goods (baskets, pottery jars, etc.) and the construction of progressively elaborate temples. This required coordination, the pooling of results of this labor (farmers expected to deliver raw and processed grains, herdsmen expected to deliver milk, dairy fats, butters and cheeses, etc.), storage and annotation of surpluses, and distribution of the resulting goods from the central store (rations). This is evident in the changing construction of houses and settlement plans, with community storage in the center.

Around this redistributive function arose chiefs that combined leadership, authority, and stewardship of resources with justice, unity, and the resulting power to mobilize and direct labor (often displayed through constructivion of prestige buildings, temples, ziggurates, etc.) as well as temples and temple-workers that included priests, scribes, as well as the specialized crafts needed to build and finish buildings, storage containers, and symbolic objects.

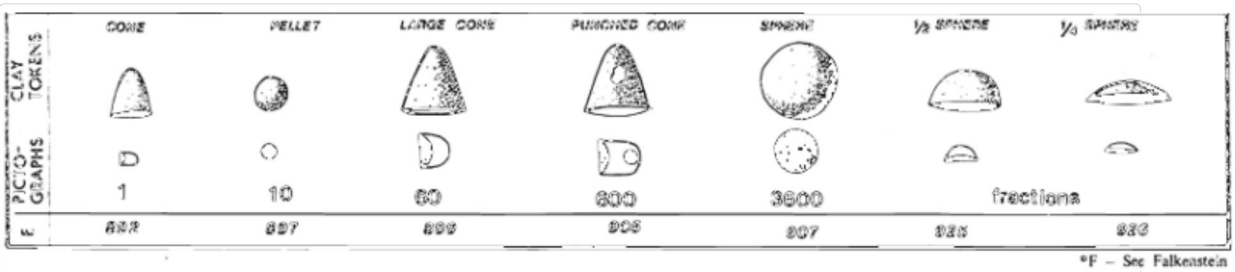
What is clear is that development of writing and mathematics in Sumeria around 3500 BCE was the culmination of a long period of increasing social and cultural complexity that accompanied the material prosperity of increasing large settled population centers at the end of the Ubaid period. The resulting mathematics was a reflection of this complexity and long heritage, as can be seen from the documented evidence of at least half a dozen metrological systems, each with its own factor list and signs. See

[**Englund/1998**, pp.30-44] and [**Hoyrup/1991**] for cultural context behind these developments.

## 2. Signs and Tablets

### 2.1. Tokens for early accounting.

The early quantitative notation grew out of a practice of accounting using clay tokens of different shapes to designate fixed measures of designated commodities using fixed metrology tables, possibly using counting boards. How extensive this token system was is not known, but archaeological and anthropological evidence shows use of pebbles (stones, abzu) by shepherds to keep track of herds, and the use of such in separate containers to track the gender and status of herded animals (male/female lamb, male/female adult, new birth, milk producing etc.)




Clay tokens mapped to the pictographs and numerical values assigned to them. (Source: Besserat/1977]

### 2.2. Clay Envelope

We know the context of token use with greater certainty when the tokens were contained in a clay envelope and kept as a unit, and when their contents were impressed on the surface of the envelope, using the same signs that were later explained with meaning when writing became more expressive. We know this because the same form ( *abzu*, pebbles or small stones) are documented in the same usage 1500 years later with cuneiform writing (see Oppenheim's find of the clay envelope at Nuzi). We know this anthropologically because it is still used in much the same form by shepherds in Middle East.

Example of clay bulla (envelope) holding clay tokens (MS 4636, found at Umma, dated to 8500-3500 BCE)

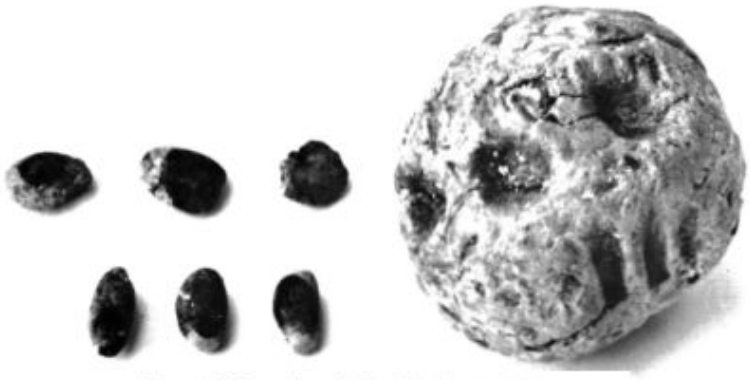


Figure 8. Susa, Iran. Bulla with its contents:
3 discs and 3 elongated pellets.
Note the corresponding marks on the outside of the bulla.

Clay tokens and bulla (envelope). Note the impressions of the tokens on the surface of the bulla (Source: Besserat/1977, SMS 1, p53

A special find at Nuzi by Leo Oppenheim dating from 1500 BCE, when cuneiform script was already advanced, confirms the usage hypothesis. On this particular clay envelope is inscribed a detailed cuneiform description of the meaning of the 48 *abzu* (small stones) inside representing 48 individual small cattle (sheep and goats).

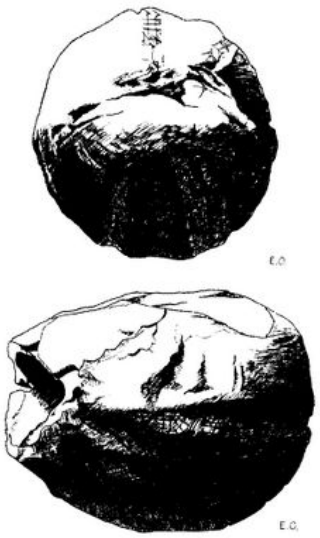


Fig. 1

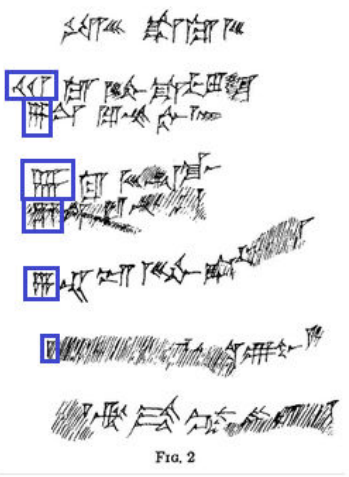


Fig. 2

Stones
"*Abnū* (referring to sheep and goats):
21 ewes that have given birth
6 female lambs
8 full grown rams
4 male lambs
6 she-goats that have given birth
1 he-goat
2 female kids
seal of Ziqarru

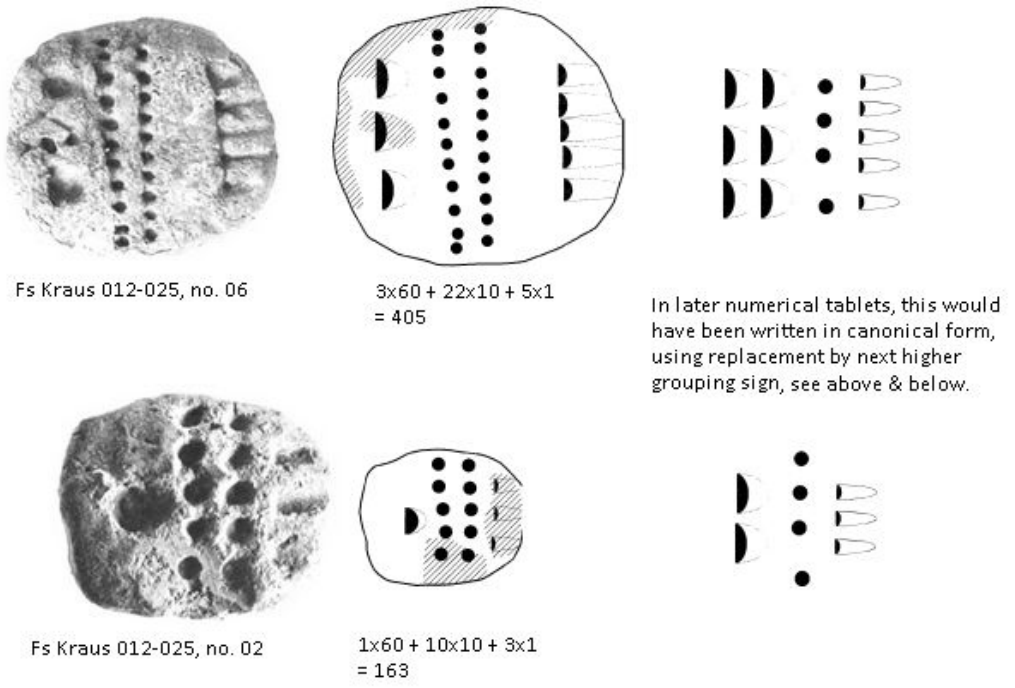
Bulla (clay envelope) with 48 tokens found at Nuzi site dated from 1500 BCE after cuneiform writing was fully developed, with an inscription describing the meaning of the tokens inside. (Source: Oppenheim/1959]

### 2.3. Early Numerical tablets (Uruk V, 3500-3350 BCE)

The next three stages can be observed in tablets dating from the **Uruk V period** from 3500 BCE to 3350 BC. Although many of these tablets come from Uruk, they cannot be dated into precise periods as they were discovered in large rubbish heaps where they had been cast aside as detritus. Some were "recycling" as building filler (and discovered in building remains). The dating (in many cases sequencing) comes from finding similar symbolism in Susa and other sites in situ amidst distinct archaeological layers (cf. **Englund/1998**, p.56). (See **Appendix 3** for timelines).

At this stage, canonical representation had not yet become standardized, i.e. the collection of quantity grouped into the largest units, i.e. the equivalent of the remainder theorem, as became standard numerical representation in later tablets. This occurs in the late numerical tablets.

Examples:




- Early numerical tablets from Jebel Aruda, c.3500-3350 BCE (Uruk V period). Notice that grouping operations to represent quantitites in canonical form (smallest possible remainder) is not yet standard. Source: **Englund/1998**, p.51, Fig.13. Tablets Fs Kraus 012-025, **no. 6 (top)** and **no. 2 (bottom)**

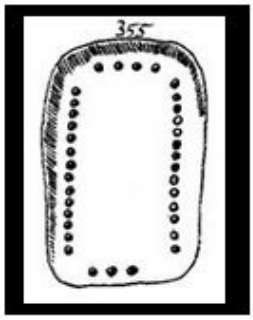
- **Thirteen additional attestations from Jebel Aruda** described by G. van Driel (1982).

### 2.4 Early Geometric Calculation

From Susa, we have what appears to be early geometric calculations, dated from Uruk V period, i.e. 3500-3350 BCE.


Field dimensions, 50 x 40

Tablet: MDP 17, 264
Susa 3500-3350 BCE




Trapezoidal dimensions:
40 and 30 are top and bottom lengths (base)
140 height

Tablet MDP 17,355
(Susa, 3500-3350 BCE)

9 (top row) x 8 (left col)

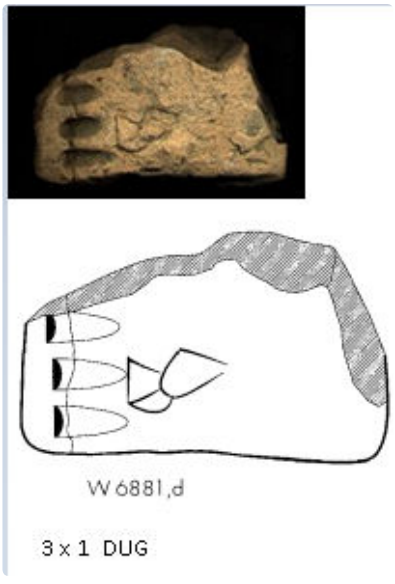
Tablet MS 3140

**Tablet MDP 17,264**. Presumably giving length measures to calculate area.

We also see in this time triangular tablets (**MW 0188/107**) and circular tablets (**MW 0188/112**, the shape which may be indicative of area or circumference records.

### 2.5 Simple Numero-Ideographic tablets, single information cell

These tablets were found as late as Uruk V period, i.e. 3500 – 3350 BCE, counting sheep (UDU) and jars (DUG) of liquid.




**Tablet W6881**,d numero-ideographic tablet from Uruk (Warka) in Uruk V period (c.3500-3350 BCE). 3 x 1 DUG (jar).

Source: **Englund/1998**, p.54, fig.16c

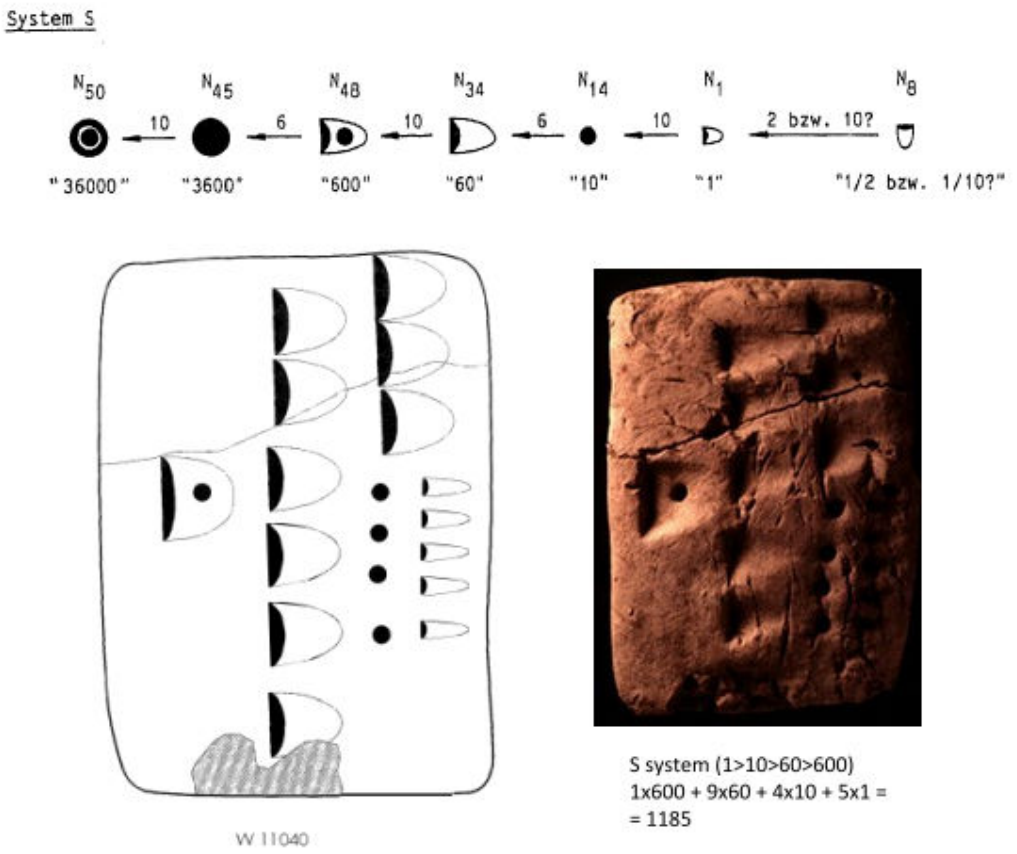
From Godin Tepe in Iran, c.3500-3350 (Uruk V period), we have the following.


**Tablet Iran 13, 9:2**, numero-ideographic tablet from Godin, Tepe in Susiana from period Uruk V (c.3500-3350 BCE), showing 3×10 + 3×1 DUG = 33 jars, with an economic cylinder seal impression. Source: **Englund/1998**, p.54, fig.16d

### 2.6. Late stage simple numerical tablets

The next phase is during the **Uruk IV period** from 3350 BCE to 3200 BCE, there is a rise in complexity of information captured in tablets, and what commodities are recorded.

What is significant is that by now the canonical representation of large numbers was standard, i.e. a sort of remainder theorem (making change type algorithm) was applied, so that repetition of smaller signs never exceeded the limit that would allow grouping and replacement/substitution with the next larger unit in the metrological sequence. Example: 6 x 10 (u) = 1 x 60 (ges), so the maximum number of tens (u) units is 5. Similarly 10 x 60 (ges) = 1 x 600 (gesu) so the maximum number of 60s (ges) units is 9. See above, where this is the case for both the 1s (dis) and the 60s (ges).



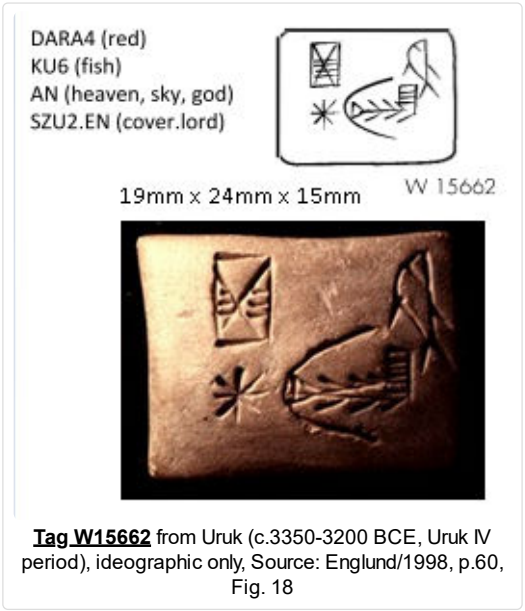

**Tablet W11040** – numerical tablet from Uruk (Warka), layer Uruk IV (c.3350-3200 BCE), showing numerical signs made from impressed tokens and a stylus for the round impressions, apparently using the sexagesimal system (S). What was being counted is not recorded, and presumably it was obvious in the context of the transaction/record. Notice the canonical representation of large numbers grouping using next largest unit of the metrology sequence so that sign repetition was minimized. Source: **Englund/1998**, p.52, fig.15

Using the **Sumerian words for numbers**, this might have been said: “gesu (600) dis (1) ges (60) ilimmu (9) u (10) limmu (4) dis (1) ia (5)”, or if the places were assumed, “dis (1) ilimmu (9) limmu (4) ia (5)”, with whatever object was being counted.

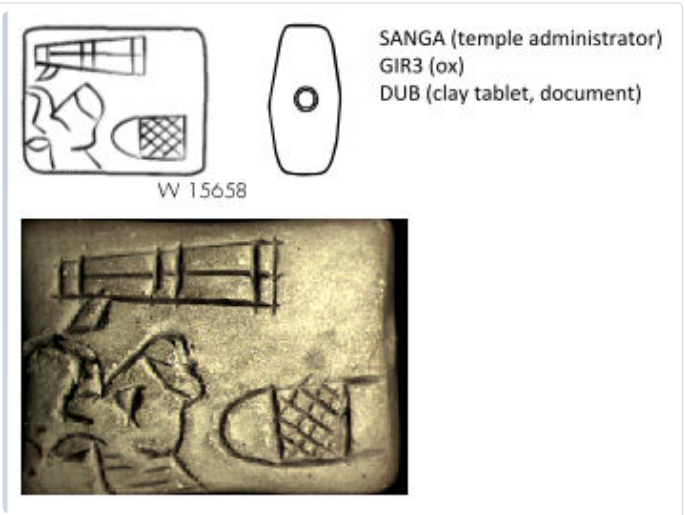
### 2.7. Simple tablets, but expanding ideographic repertoire

Texts began to capture more than just quantity, but also other details through additional signs: commodity, ownership, use function. Example: **127 finds from Uruk (Uruk IV period 3350BCE to 3200 BCE)**. Tags were solely ideographic. Tablets combined number with attributes, not all of which have been deciphered, sometimes with what appears to be signature of an individual connected with the transaction.

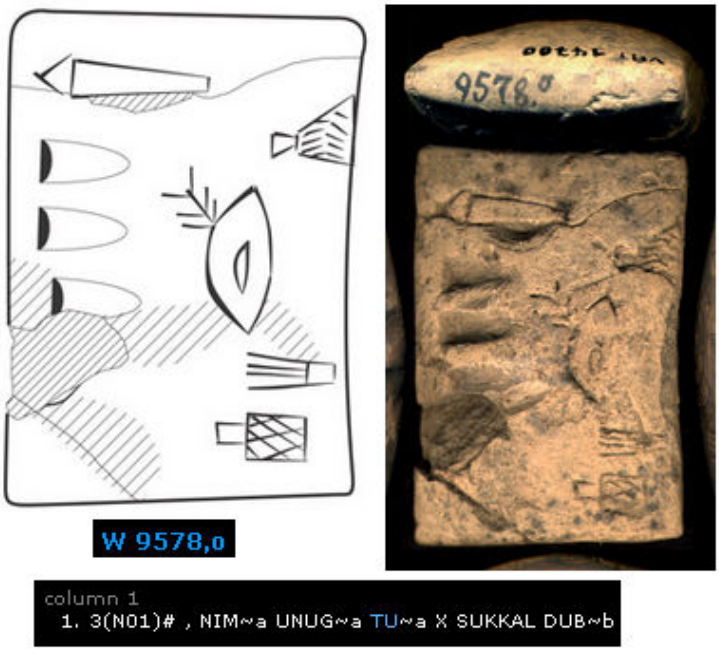
Examples:



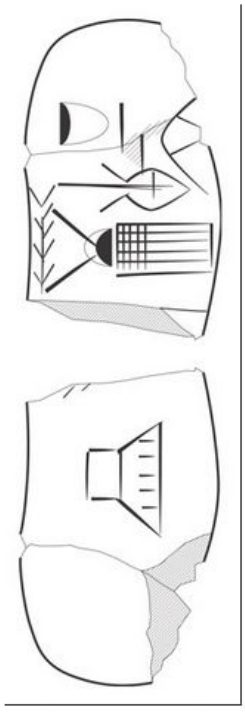

**Tag W15662** from Uruk (c.3350-3200 BCE, Uruk IV period), ideographic only, Source: Englund/1998, p.60, Fig. 18




**Tablet W15658** from Uruk (3350-3200BCE, Uruk IV period), ideographic, Source: Englund/1998, p.60, Fig. 18




**Tablet w9478,o** from Uruk (Uruk IV period, 3350-3200 BCE). This is an example of an ideographic tablet conveying a great deal more information about the context of the transaction (conj. transl. "high messenger from Uruk records this document") than simply the quantity three. Unfortunately, we don't know what the three refers to.



obverse

column 1
1. 1(N34) , TAR~a# AB2# GIR2~a |SILA3~axSZE~a@t| E2~b
2. [...] , [...]

reverse

column 1
1. , [...] SZE~a#? UNUG~a

60 (to cut) (cow) (knife) (sila of barley) (house)
(barley) (Uruk)

W 15897,c24

- **Tablet w15897, c24** from Uruk (Uruk IV period, 3350-3200 BCE). This tablet describes a quantity of prepared food intended for a house. The reverse (Uruk) might be the origin or destination, source, or recipient.

Of course, any attempt at translation is hypothetical, pending similar sign groupings or corroboration with later texts or other contextual implements. But this serves to illustrate the change that started to occur with the broader use of ideograms.

See **Appendix 1** for proto-cuneiform (archaic) sign lists and **Appendix 4** for primary source research tools to decipher/verify transliteration/translation of proto-cuneiform primary source texts.

### 2.8. Complex Numero-ideographic tablets, with multiple information cells

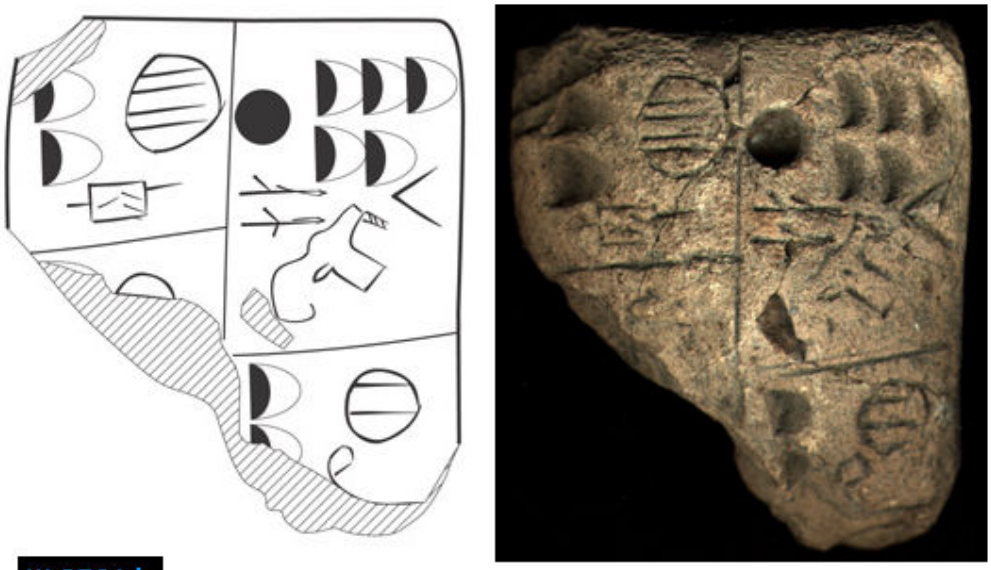
This is the scribal equivalent of a spreadsheet, containing lists of multiple items with quantity and commodity, and sometimes attributes. Proto-cuneiform from the Uruk influence listed the number first. Proto-elamite from the Susa influence listed the commodity first.

The primary administrative activity in archaic Mesopotamia was of grain storage and distribution, and these by far have the greatest number of accounts in Uruk (Englund/2001,p.3)

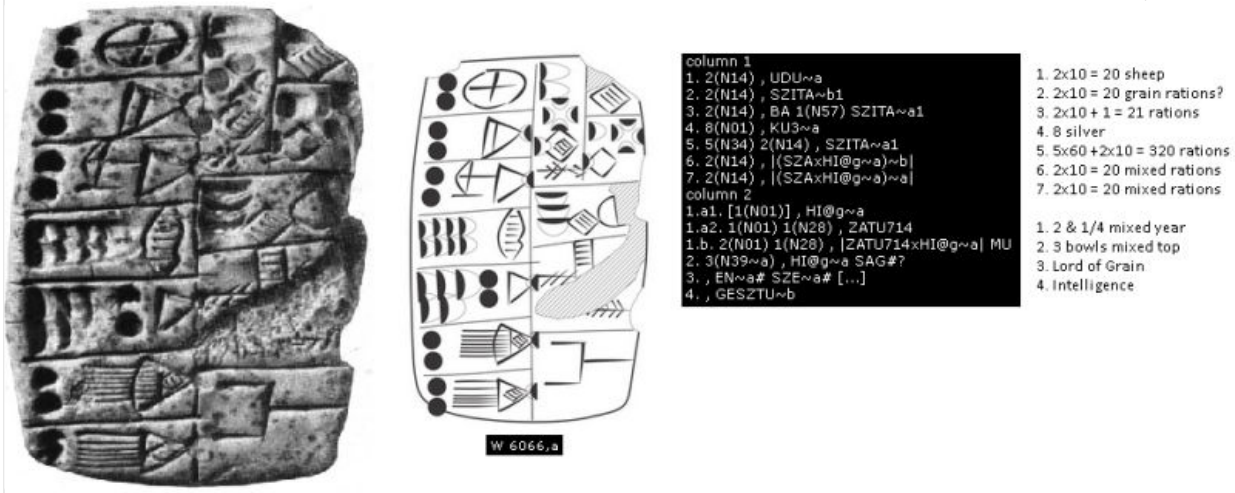
There are accounts of:

- grain and grain products (emmer wheat, barley, breads and other baked goods, cereal products, rations),
- beer of various strengths and its primary ingredients (barley, hops, malt),
- other liquid grain products (e.g. mixed with dairy fats)
- herded animals (sheep, goats, cows, pigs) and their production (dairy fats, milk, cheese)
- land usage
- labor management, wages, and the distribution of rationed goods

These activities and their signs are found on archaic tablets from Uruk and surrounding cities in the periods Uruk IV (3350 BCE – 3100 BCE) and Uruk III/Jemdet Nasr (3100 BCE – 3000 BCE), and indeed appeared in the period Uruk V (3500-3350 BCE) in early attestations.




**Tablet W9761,b** from Uruk (Uruk IV period, 3350 BCE -3200 BCE). This tablet appears to describe clothing from wool, but more interestingly, apparently the making of surface or outer wear (TUG2 MUSZ3) from deer hides(?) (DARA3xKAR2).




**Tablet w6066,a** from Uruk (in Uruk IV period, c.3350 – 3200 BCE).

### 2.9. Double-sided tablets

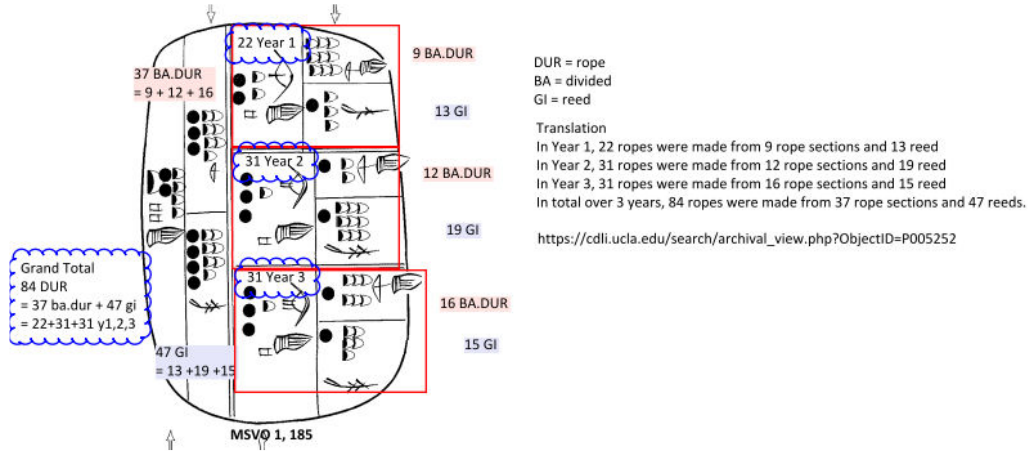
The first double-sided complex numero-ideographic tablets were seen during Uruk IV (3350 BCE-3200 BCE). It is unclear whether and how the information on the reverse was in every case related to that on the obverse (front), and how much was fixed by convention vs. varied by tablet/context. Could the double-sided documents show the two trades on separate sides? Was the reverse in some cases a remainder after settling a transaction (i.e. input/output)? What we know is that in many cases the reverse was a high-level summation (grand total) operations over all quantities provided with detailed accounting on the front. (cf. Englund/1998, p.61.ff)

Examples

- **Tablet W7227,a** which contains 54 cows (AB2) and bulls (GU4), the largest attested herd of cattle in Uruk IV period.
- **Tablet IM 074343**, which looks like what might be traded on one side (complex products: 10x jars

of beer, 25x fruits, 40x apples, 4x special fruits, 3x foreign or exotic fruits, 5x luxurious fruits or almonds, 2x apple fruit, 15x wool, 71x ?, 2x onion/garlic, 3x perfume) with the counter trade on the reverse (simple raw materials: 20x onions/garlic, 20x blocks/slabs of stone, 16x boxes of fish.

- **Tablet W6966,b** looks like a wage distribution receipt for 20 male laborers (GURUSZ) receiving 31 wage rations (BA).
- **Tablet MSVO 1,185** (from Jemdet Nasr during Uruk III period) is a 4-column account of total rope (DUR) production over three years from two production inputs (column 4): pre-existing rope pieces (BA.DUR) and fresh reed (GI). A yearly total is given in column 3, an subtotal of each individual input (just rope pieces and just reed separately) over all three years (column 2), and then finally the grand total of new ropes is given in column 1, obtainable by summing either column 2 or column 3.




**Tablet MSVO 1,185** during Uruk III period (3200-3000 BCE) shows detailed accounting of the production of ropes (DUR) from inputs pre-existing pieces (BA.DUR) and new reed (GI). Source: **Englund/1998**, p.63

## 3. Mathematical Capability

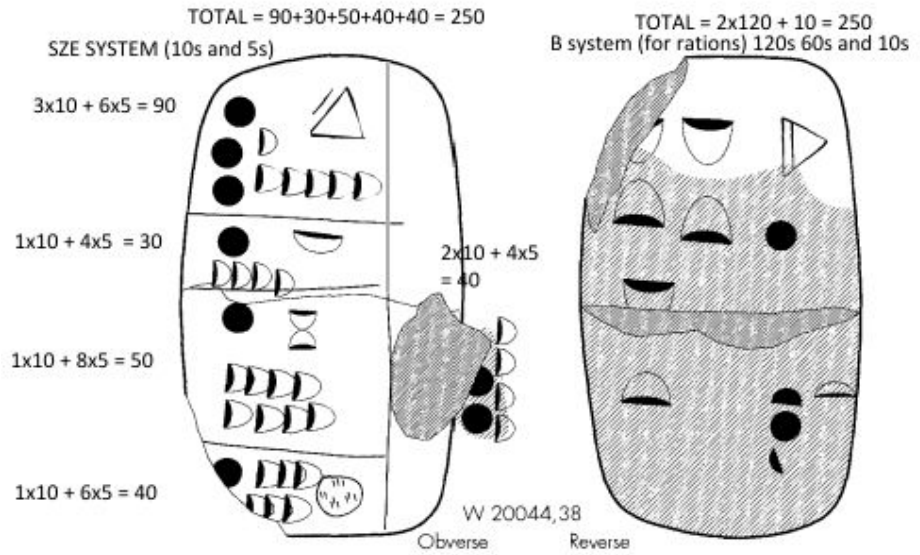
In what follows, we will look at some of the mathematics evident from the early archaic tablets.

### 3.1. Simultaneous Metrology Systems

One of the complexities of the Sumerian measurement system was a set of conventional measures that had different units based on what commodity was being measured. This meant almost a dozen parallel metrology systems were in simultaneous use, some using the same signs, but with different values, sequences, and factors between them.

The following tablet illustrates the commodity specific context behind the use of metrology systems even on the same tablet.

**Example: Uruk IV use of SZE and B systems on same tablet**




**Tablet W20044** from Uruk (Uruk IV period, 3350-3200 BCE) showing simultaneous use of SZE (grain) system and B (rationed foods) system for counting. Source: **Englund/1998**, p.62

**Metrology systems**

Looking at the metrology sequences in use, the most common systems were the sexagesimal (S system) for counting discrete objects, bisexagesimal (B system) for counting rationed goods, SZE system for counting grain capacity, and the GAN2 (G) system for measuring area. The S system progresses by factors of 10 and 6, the B system appends a factor of 2 after the 10 and 6, the SZE system has a completely different sequence 5, 6, 10, 3, and the G system reverses the last two with 6, 3, 10. This means the same symbol means 1 unit (dis) in the S and B systems but 5 sila (bowls) in the SZE system, and 1 iku in the G system. A small circle is then worth 10 units (u) in the S system, e.g. when counting sheep, 6 when counting barley, and 18 when measuring the area of a field. [**Nissen/1993**, p.28 and 132].

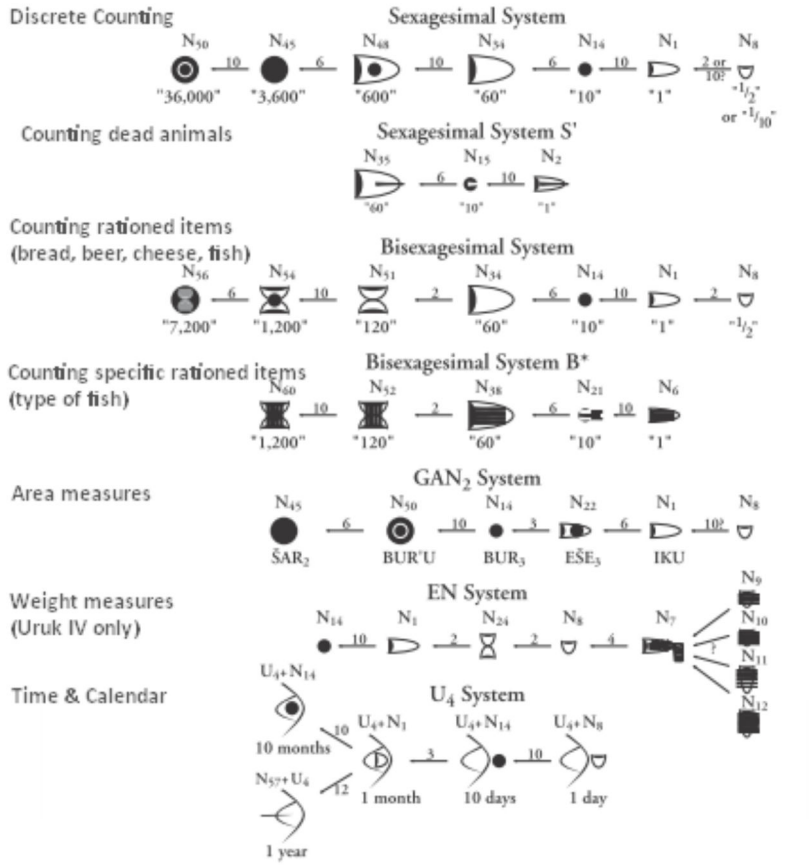
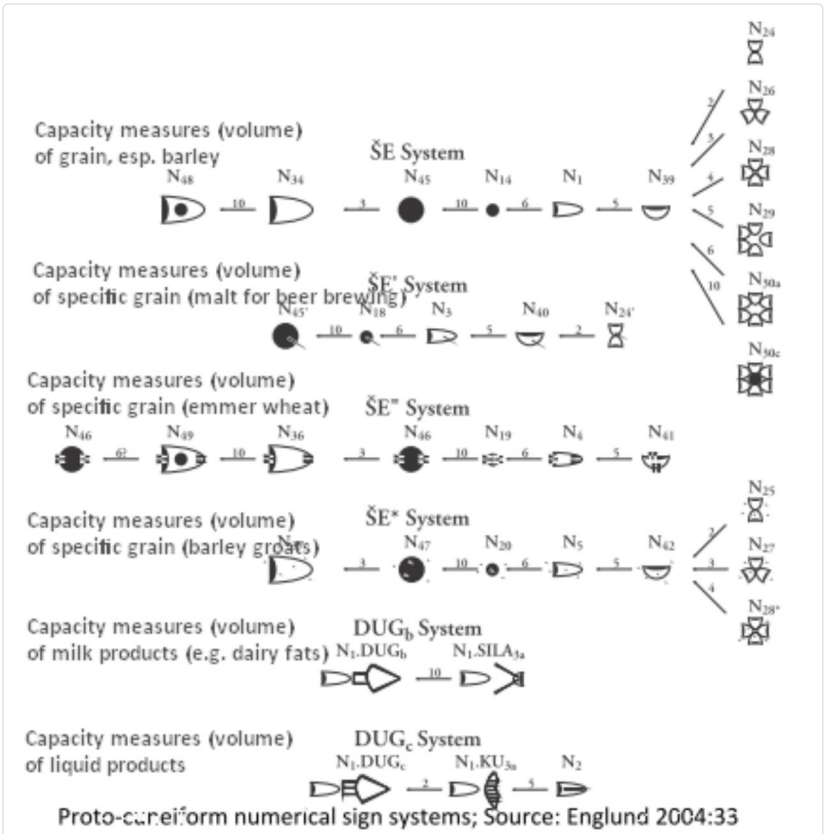



Figure 4a: Proto-cuneiform numerical sign systems.
Several systems of numerical signs served to qualify discrete objects (Fig. 4a), while others qualified measures of grains, (semi-)liquids and time (Fig. 4a and 4b).

Early metrology (counting & measurement) used separate systems depending on the commodity being measured. (Source: **Nissen/1993**, pp.28-29, **Englund/2004**, pp.32-33)




Early metrology (counting & measurement) used separate systems depending on the commodity being measured. (Source: **Nissen/1993**, pp.28-29, **Englund/2004**, pp.32-33)

The factors of the above systems are:

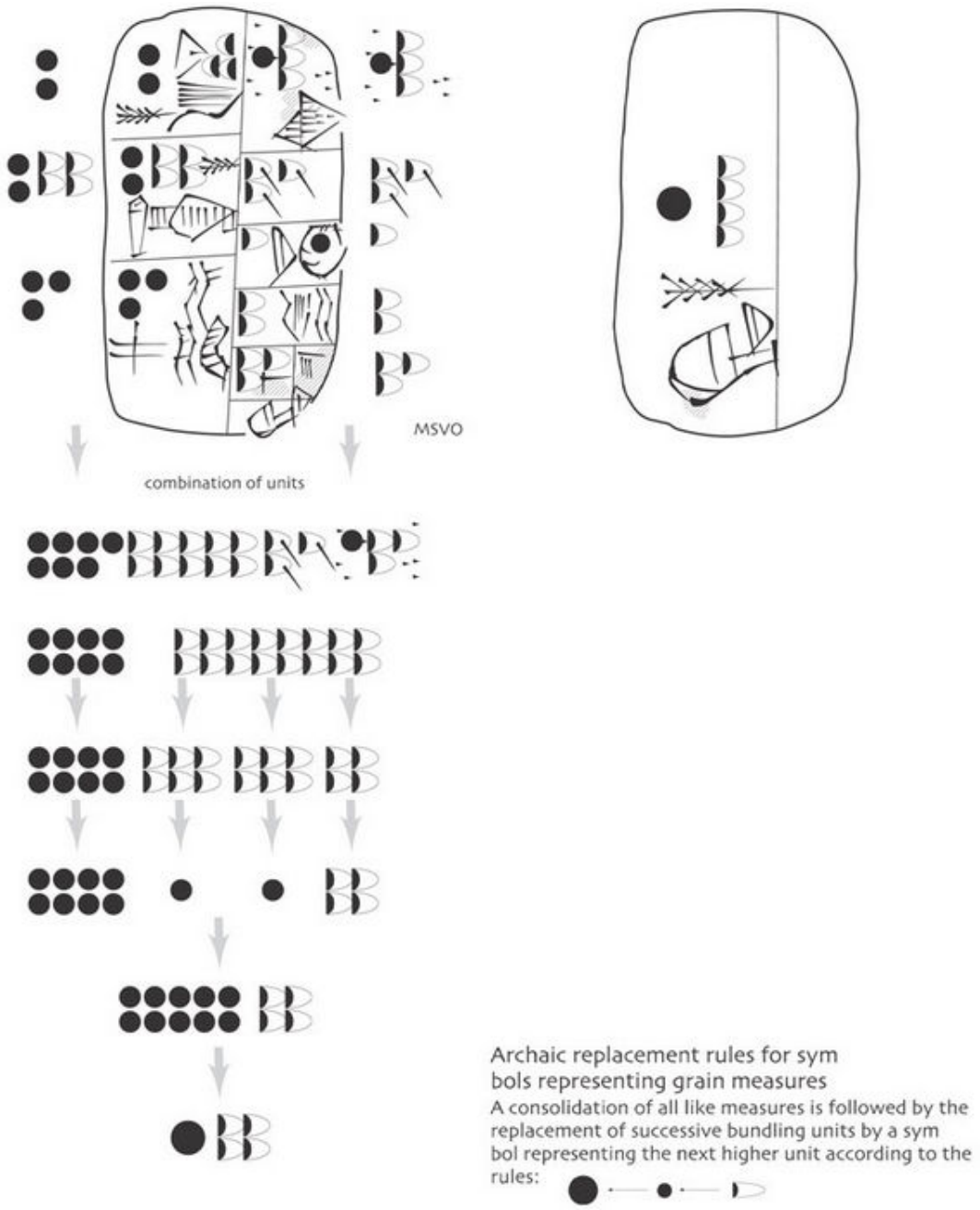
- sexagesimal series (for general counting) with alternating factors of 10x and 6x e.g. 1, 10, 60, 600, 3600, 36000, …,
- bisexagesimal series (for counting rationed products) with factors 10x, 6x, 2x e.g. 1, 10, 60, 120, 1200, 7200, …,
- sze series (for grain capacity) with factors 5x, 6x, 10x, 3x, and all unit fractional designations from 1/2, 1/3, …, to 1/10, e.g. 1, 5, 30, 300, 900, 9000, …
- dug series (for liquid capacity) with factors 5x, 2x, e.g. 1, 5, 10, 50, 100, …
- gan series (for area masures) with factors 10x, 6x, 3x, 10x, 6x, 3x, … e.g. 1, 10, 60, 180, 1800, 10800, 32400, …
- en series (for weight measures) with factors 4x, 2x, 2x, 10x, e.g. 1, 4, 8, 16, 160, …
- u series (for time and calendar) with factors 10x, 3x, and 12x, corresponding to 1, 10, 30, 360 (day, 10-day week, month, and year).

**3.2. Arithmetic – Sums with Grouping and Replacement**

From Uruk III, complex tablets had detailed accounting on the front (obverse) and a simple sum tally of the higher level related items, on the back (reverse). As we have seen above, this involved grouping and replacement using the appropriate metrological factors depending on what was being counted.

Tablets from Jemdet Nasr (MSVO 1) in N. Mesopotamia (an economic outpost perhaps of Uruk) cover broader aspects of the archaic provincial economy for which accounting was used, including herding, land management, utilization, and yield planning, worker rationing, and other distributive mechanisms. These show the more detailed accounting practice.

Example: MSVO 1,216 (Nissen/1993, p.133)

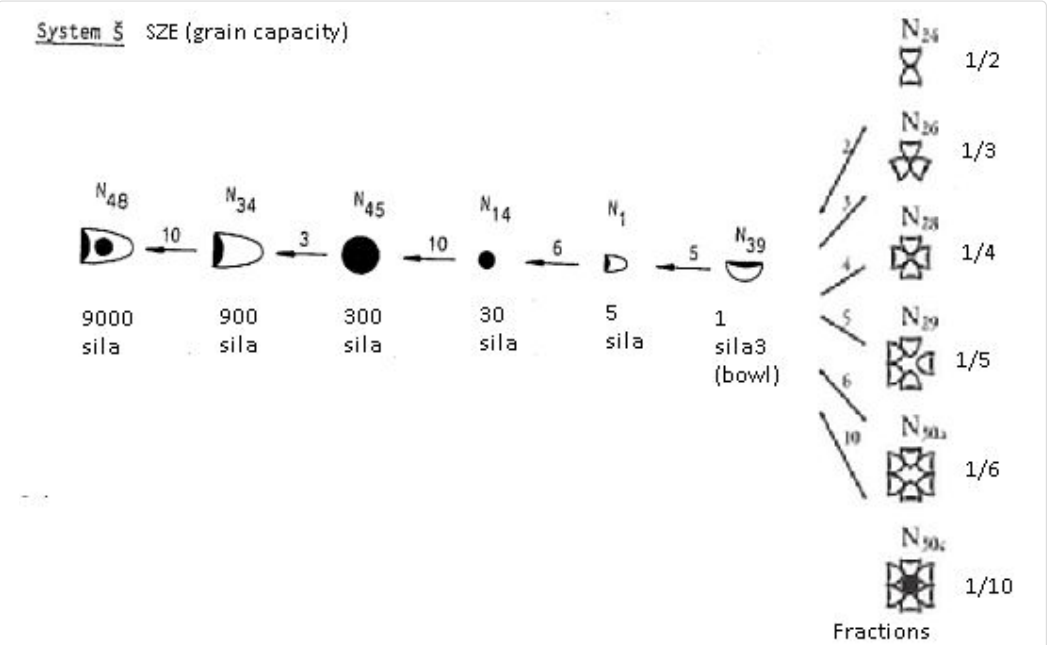



**Tablet MSVO 1, 216** from Jemdet Nasr (Uruk III period, 3200-3000 BCE). Shows double-sided tablet with detailed accounting on the front and a grand total of the high level items on the reverse, using grouping and replacement. Source: Nissen/1993, p.133

It is unclear whether the early scribes used a wooden counting board to perform the arithmetic/groupings (see Christine Proust's reconstructions).

### 3.3. Arithmetic – Fractions and Multiples

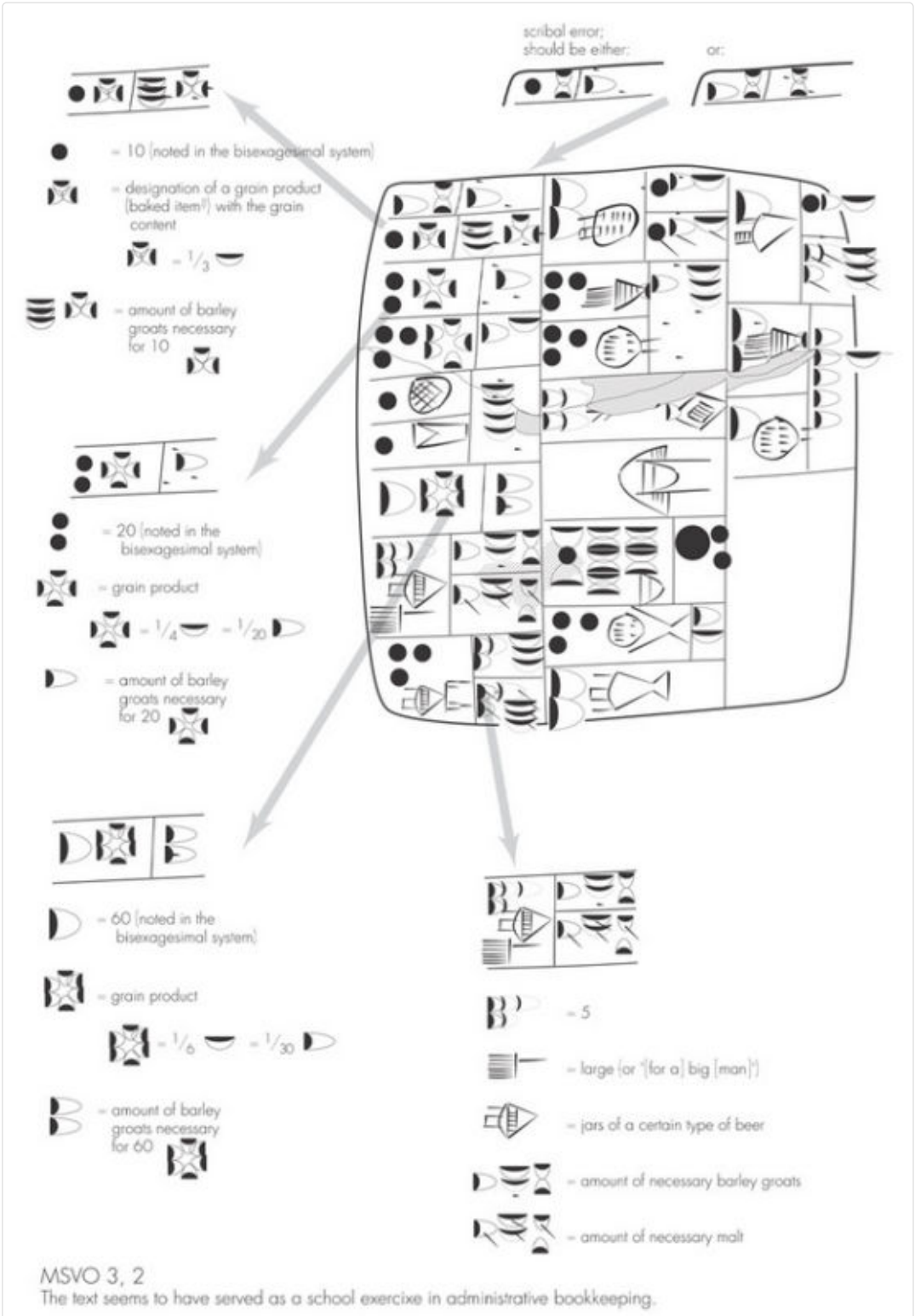
The SZE system for measuring grain capacity is where we see the use of fractions.




SZE system for measuring grain capacity in units of 1 sila3 (bowl). Notice the fractions.

The example below is from the Uruk III period (3200-3000 BCE) from the Erlenmayer collection of archaic tablets (MSVO 3), which appear to have been an administrative archive of a production unit concerned with the distribution of beer and the ingredients used in beer brewing (unprocessed grain emmer and barley, malt, coarse-ground barley groats). The tablets in this collection document production processes, e.g. how much grain and malt was needed to produce a certain type, size, and strength of beer.


**Tablet MSVO 3, 02** from Uruk (in Uruk III period, 3200-3000 BCE), showing a complex numero-ideographic tablet detailing grain products and the raw materials required to manufacture them.



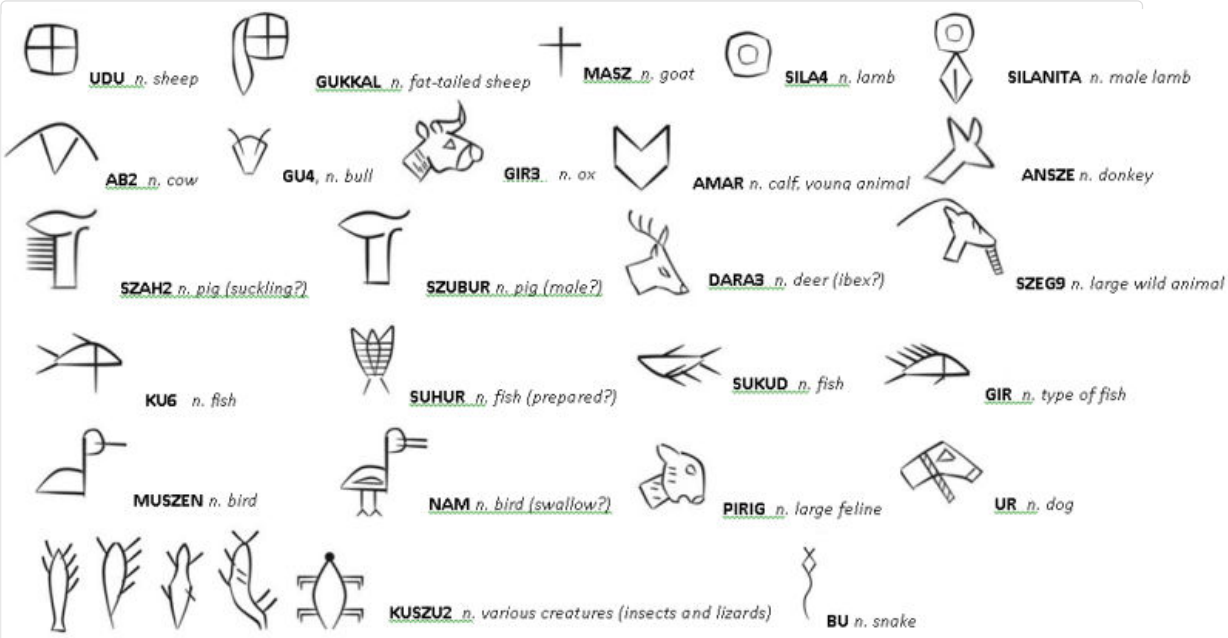

Interpreting pictographs in early accounting texts. This tablet, formerly from the Erlenmayer Collection appears to have been part of the administrative archive of a production unit concerned with the distribution of beer and the ingredients used in beer brewing (unprocessed grain emmer and barley, malt, coarse-ground barley groats). The primary administrative activity in archaic Mesopotamia was of grain storage and distribution, and these are by far have the greatest number of accounts in Uruk. (**Englund/2001**,p.3) (Source: Tablet **MSVO 3, 02** (3 columns). **Interpretation**. Publictions: **Nissen/1993** frontspiece, p.42, **Englund/1998**)

***Download article (PDF)***

## Appendices

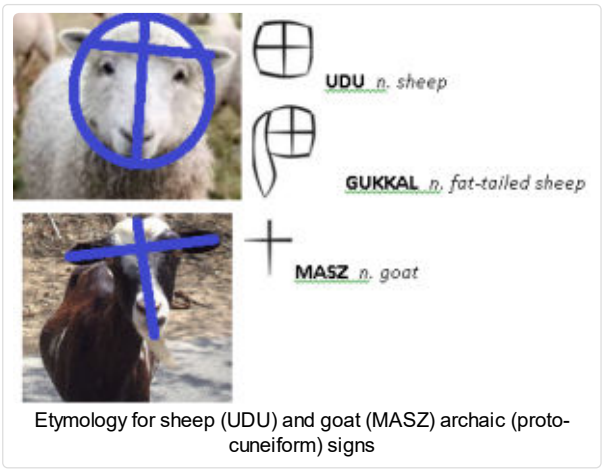
### Appendix 1. Proto-Cuneiform (Archaic) Word Signs (Vocabulary)

**Table 1. Animals**



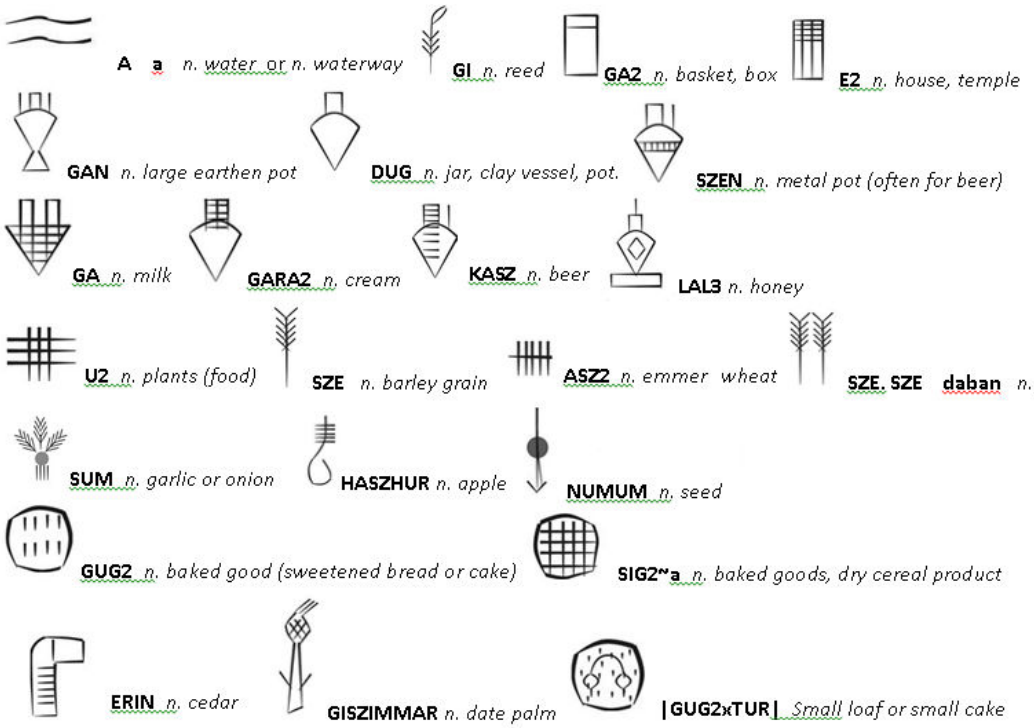

Sign List, animals in the neolithic economy. Note, all signs have been rotated 90 degrees clockwise to conform to how they would have been read, vs. the Assyriological convention for how they are presented.

**Etymology of animal signs** is clear for most just by looking. Except perhaps **sheep** and **goat**. Why are sheep (UDU) a cross within a circle, and goat (MASZ) a cross (no circle)? Imagine looking at a sheep head on. Plump wooly head (circle), with the vertical axis defined by broad nose ridge and horizontal axis defined by the extended ears. With goats, no wool but same distinctive cross axes for the face (cross-without circle). With **fat-tailed sheep** (GUKKAL, a distinct type of sheep, 25% of world sheep population), the fat tail (up to 16% of sheep's weight, concentrated in the tail and therefore easy to harvest as a source of cooking fat/tallow) is pinned to the back of the symbol.



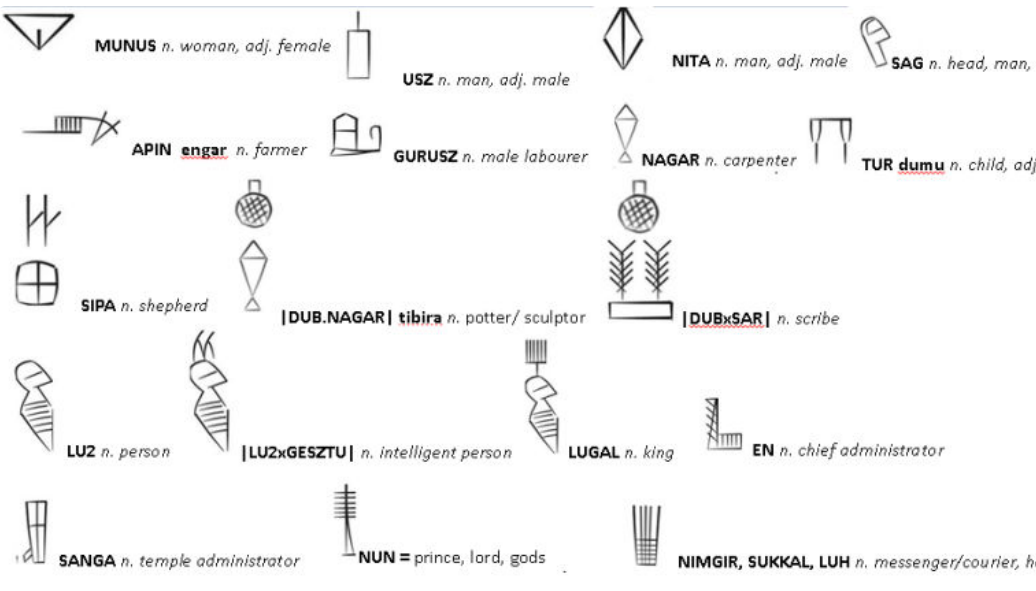

Etymology for sheep (UDU) and goat (MASZ) archaic (proto-cuneiform) signs

**Table 2. Grain and Grain products, Beverages, Jugs of Liquid**




Sign List, beverages, grain, and grain products in the late neolithic economy. Note, all signs have been rotated 90 degrees clockwise to conform to how they would have been read, vs. the Assyriological convention for how they are presented.

**Table 3. People and Professions**



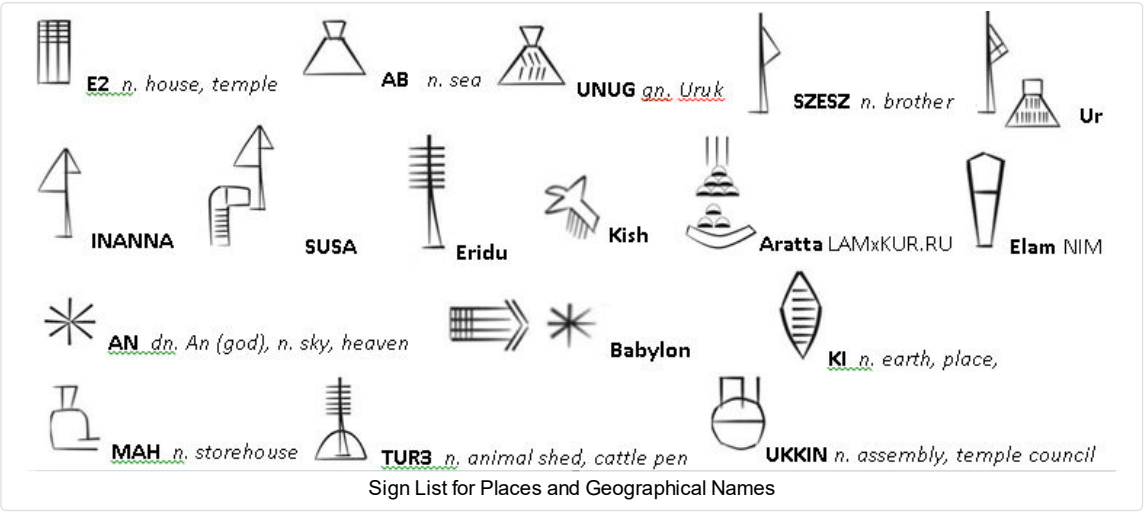

Sign List for people and professions

**Etymology for People & Professions**

- The origin of signs for male and female should be pretty obvious just by looking.
- Farmer (ENGAR) is from lord (EN) and a grain product (GAR) and uses the sign of a plow (APIN).
- Male laborourer (GURSZ) is from GUR (referring to various forms of labor and the largest form of capacity measure e.g. for grain, 1 gur = 300 sila (approx 300 litres) [**Nissen/1993**, p.142], or 480 sila, approx. 480 litres, [**Robson/2007**, p.70]) and USZ (male).
- Shepherd (SIPA) uses the double-sign for PAP (to check, verify, count) and UDU (sheep). Note, interestingly the word UDU is from UD (day) and U (pasture).

- Potter (DUB.NAGAR) is from clay (DUB) and carpenter (NAGAR)
- Scribe (DUBSAR) is from clay (DUB) and 'to write' (SAR). The sign for SAR (to write) is, interestingly, of grain (SZE) on a tablet, perhaps a reference to the bookkeeping motivation for accounting in the first place.
- Intelligent person (LU2xGESZTU) is from LU2 (person) and GESZTU (ears, intelligence). Note that the sign for GESZTU is the same for ears, suggesting that intelligence is associated with listening well.
- King (LUGAL) is from LU2 (person) and GAL (large or great).
- Chief Administrator (EN) is common first part of names of rulers, e.g. En-mer-kar, En-men-bara-ges-i, En-men-lu-an-na, En-men-gal-an-na, En-sipa-zi-anna, En-men-nun-na, En-nun-dara-an-na, En-shakush-an-na, En-bi-esztar. MER is crown, BARA2 is pedestal, LU2 is person, AN is god, GAL is great, SIPA is shepherd, NUN is prince or god.
- Herald (NIMGIR) is from NIM (high) and GIR (fish)
- Child (DUMU) uses the sign TUR (small)

**Table 4. Places and Geographical Names**



Sign List for Places and Geographical Names

**Etymology for Place Names**

- Eridu has the the sign NUN (prince or god).
- KI is place/earth. According to Sumerian mythology, the god ENKI from EN (administrator) and KI (earth) is the god that cared for mankind enough to teach civilizations, and first taught these arts in Eridu.
- Uruk (UNUG) has the sign sea (AB) with many people
- Ur has the signs SZESZ (brother) and Uruk (UNUG)
- Kish, interestingly, has the sign donkey (ANSZE) with many people. Kish is in N. Mesopotamia (Akkad) and as a major trading area of its own would have been closely associated with donkeys.
- Babylon is the signs KA2 (gate) and AN (god, sky)
- Susa has the signs ERIN (cedar) and Inanna. Background, in the Gilgamesh cycle, it is clear that cedar was the sought-after wood from the mountains used to build the great temples, and Inanna was the patron diety (goddess) of Uruk. Susa is in the Zagros mountains, and is likely one of the main trading centers of Uruk in the Uruk Expansion period (Uruk IV).
- Aratta is from the signs LAM (abundance), KUR (foreign, i.e. over the mountains, these would be the Zagros mountains to the East of Mesopotamia), and RU (to cast down, i.e. dominate). See literature Uruk Cycle: **Enmerkar (founder of Uruk) and the Lord of Aratta**
- Elam has the sign HIGH

- Storehouse is a vessel from which flows out.
- Animal shed (TUR3) has the signs NUN (prince) and cover (SZU2, DU6)

**Figure 5. Map of Places in the Ubaid/Uruk periods**


Near East ca 4300 BCE toward the end of the Ubaid period and before the earliest discovery of accounting. Notice the many city-states that had arisen in the alluvial flood plains between the Tigris and Euphrates rivers. In particular, Uruk and Susa would rise as prominent city-states.

## Appendix 2. Sumerian Language (Emegir)

### Sumerian Nouns – Table 1

**Sumerian Nouns** ** indicates compound noun word; sz is pronounced like Zsolt (sound not in English)*

| | People | | Animals | | Produce | | Places | |
|---|---|---|---|---|---|---|---|---|
| 1 | person | lu2-ulu3 | 1 sheep | udu* | 1 bread | nin-da* | 1 house | e2 |
| 2 | man | lu2 | 2 lamb | sila4 | 2 water | a | 2 town/city | iri |
| 3 | woman | munus | 3 goat | masz | 3 milk | ga | 3 plain | e-den |
| 4 | father | ab-ba | 4 cow | ab2 | 4 cheese | ga-ar3 | 4 field | gana2, a-szag* |
| 5 | mother | ama | 5 ox | gu4/gud | 5 pulses* | gu | 5 sea | ab |
| 6 | brother | ses | 6 donkey | ansze | 6 barley | sze | 6 river | id2 |
| 7 | sister/lady | nin9 | 7 fish | ku6 | 7 wheat (emmer) | gig (ziz) | 7 mountain/foreign land | kur |
| 8 | wife/husband | dam | 8 bird | muszen | 8 flour | zid2, dabin | 8 temple | e-ku3* |
| 9 | child | du-mu* | 9 duck | us | 9 seed | numun | 9 palace | e-gal* |
| 10 | son | dumu-nita2* | 10 chicken | <none> | 10 eggs | nunuz | 10 pasture/green meadows | u2-sal* |
| 11 | daughter | dumu-munus* | 11 pig | szah2 | 11 honey | lal2 | 11 wall/fortification | bad3 |
| 12 | king | lu-gal* | 12 dog/wolf | ur | 12 dates | zu2-lum | 12 irrigation canal | par (pa 5/6) |
| 13 | high priest, lord | en | 13 cat | <none> | 13 apple | haszhur | 13 well (water well) | pu2 |
| 14 | governor | en-si2 | 14 horse | <none> | 14 oil | i3 | 14 sky/heaven | an |
| 15 | shepherd | si-pa.d* | 15 lion | ur-mah* | 15 ghee (butter, oil) | i2-nun | 15 sun | ud |
| 16 | fisherman | szu-ku6* | 16 snake | uszum | 16 sesame oil | i3.gisz | 16 moon | usakar |
| 17 | farmer | engar | 17 dragon | uszum-gal | 17 wool | siki | 17 star | mul |
| 18 | scribe | dub-sar* | 18 gazelle | masz-a | 18 malt | munu4 | 18 earth, place | ki |
| 19 | merchant | dam-gara | 19 fox | ka5-a | 19 salt | mun | 19 forest | tir |
| 20 | cook | muhaldim | 20 bear | aza | 20 herbs | szim | | |
| 21 | carpenter | nagar | | | | | | |
| 22 | musician | nar | **Gender** | | **Shapes** | | **Time** | |
| 23 | judge | di-ku5* | 1 male | nita2 | 1 circle | s˜ar2 | 1 day | u4.d |
| 24 | coppersmith | tibira | 2 female | munus | 2 outside | bar | 2 night | gi6 |
| 25 | potter | bahar | | | 3 inside (heart) | s˜a3.g | 3 week | <none> |
| 26 | weaver | us.bar | | | 4 square | sa2 | 4 month | iti |
| 27 | sage, priest | ab2-gal*, umannu (apkallu Akkadian) | | | | | 5 year | mu |
| 28 | slave | sag (head) | | | *lentils, beans, peas | | | |
| 29 | slave man | urdu2 | | | https://pulses.org/what-are-pulses/visual-guide-to-pulses | | | |
| 30 | slave woman | geme2 | | | | | | |
| 31 | friend | ku.li | | | | | | |

Sumerian nouns, 1 (Source: **Jagersma/2010**, ePSD/2006)

**Sumerian Nouns – Table 2**

| | Things | | Body parts | |
|---|---|---|---|---|
| 1 | something | nig2 | 1 body/skin | kusz |
| 2 | clothes/cloth | tug2 | 2 head | sag |
| 3 | shoe | e-sir2 | 3 forehead | sag-ki |
| 4 | bowl | bur | 4 hair | siki* |
| 5 | pot | dug | 5 eye | igi |
| 6 | basket | hal, dusu | 6 nose | inim |
| 7 | grass | u2 | 7 mouth | ka |
| 8 | reed | ge | 8 tongue | eme |
| 9 | tree | mes, gisz˜ | 9 tooth | zu2 |
| 10 | cedar | eren | 10 ear | gesztu |
| 11 | wood | gisz | 11 neck | gu2 |
| 12 | sand | sahar | 12 chest,breast | ga-ba* |
| 13 | silver | ku-bar6-bar6 | 13 heart, belly | sza3.g |
| 14 | gold | ku-sig17 | 14 arm | a2 |
| 15 | copper | urud | 15 hand | szu |
| 16 | horn | si1 | 16 finger | szu-si |
| 17 | tablet | dub | 17 leg, foot | giri3 |
| 18 | judgement | di | 18 knee | du10.g |
| 19 | stone, gem | na4, za | 19 toe | giri3-si |
| 20 | plough | apin | | |
| 21 | wagon | mar | **Abstract Things** | |
| 22 | god | digir | 1 life | zi, nam-ti.l |
| 23 | harp | balag | 2 word | inim |
| 24 | boat | ma2 | 3 name | mu |
| 25 | street/road | sila | 4 prayer | nam-sita* |
| 26 | knife | gir2 | 5 inventory | gurum2 |
| 27 | ring | har | 6 X-ship, X state | nam-X |
| 28 | fire | izi | | |

Sumerian Nouns, 2 (Source: **Jagersma/2010**, ePSD/2006)

**Sumerian Verbs – Table 3**

**Sumerian Verbs** (Jagersma 2010:292)

| Motion | | Location | | Actions for Living | |
|---|---|---|---|---|---|
| 1 walk | | 14 be somewhe | *gal2* | 24 eat | *gu7/ku2* |
| 2 run | *kas4* | 15 place (put?) | *gar* | 25 drink | *nag* |
| 3 ride | | 16 go, come | *gen* | 26 give birth to | *du2.d* |
| 4 fly | *dal* | 17 go up or dow | *e11.d* | 27 grow | *mu2* |
| 5 fall | *sub* | 18 go out | *e3* | 28 grow up | *bulug* |
| 6 bow | *gam* | 19 go back | *gur* | 29 do, say | *du11.g, e* |
| 7 stand | *gub* | 20 approach | *te* | 30 confirm | *ge.n* |
| 8 sit | *tus* | 21 follow | *us2* | 31 write | *sar* |
| 9 lie | *nu2* | 22 surround | *dab6* | 32 give | *sum2* |
| 10 see (eye) | *igi* | 23 cross | *bala* | 33 bring, fetch | *re6* |
| 11 look at | *igi bar* | | | 34 enter, bring in | *ku4.r* |
| 12 hear (ear) | *gestu* | | | 35 carry | *ga6.g, tu* |
| 13 know | *zu* | | | 36 search for | *kig* |
| | | | | 37 find | *pa.d* |
| | | | | 38 barter, buy, sell | *sa* |

Sumerian Verbs 1 (Source: **Jagersma/2010**, ePSD/2006)

### Sumerian Verbs – Table 4

| Constructive | | Transformative/Destructive | |
|---|---|---|---|
| 39 touch | *tag* | 52 cut | *ku5.r* |
| 40 press | *sur* | 53 break | *has* |
| 41 rub | *sub* | 54 roast | *sa* |
| 42 mix | *hi* | 55 bake | *seg* |
| 43 stir | *lu* | 56 burn | *bil2* |
| 44 pour | *de2* | 57 destroy | *gul* |
| 45 fill | *si* | 58 hit | *ra* |
| 46 tie, bind | *kese2.r* | 59 strike | *sig3* |
| 47 weave | *tuku* | 60 die | *us2* |
| 48 dig | *ba-al* | | |
| 49 cover | *dul, su* | | |
| 50 make | *ak* | | |
| 51 erect, plant, build | *ru2* | | |

Sumerian Verbs 2 (Source: **Jagersma/2010**, ePSD/2006)

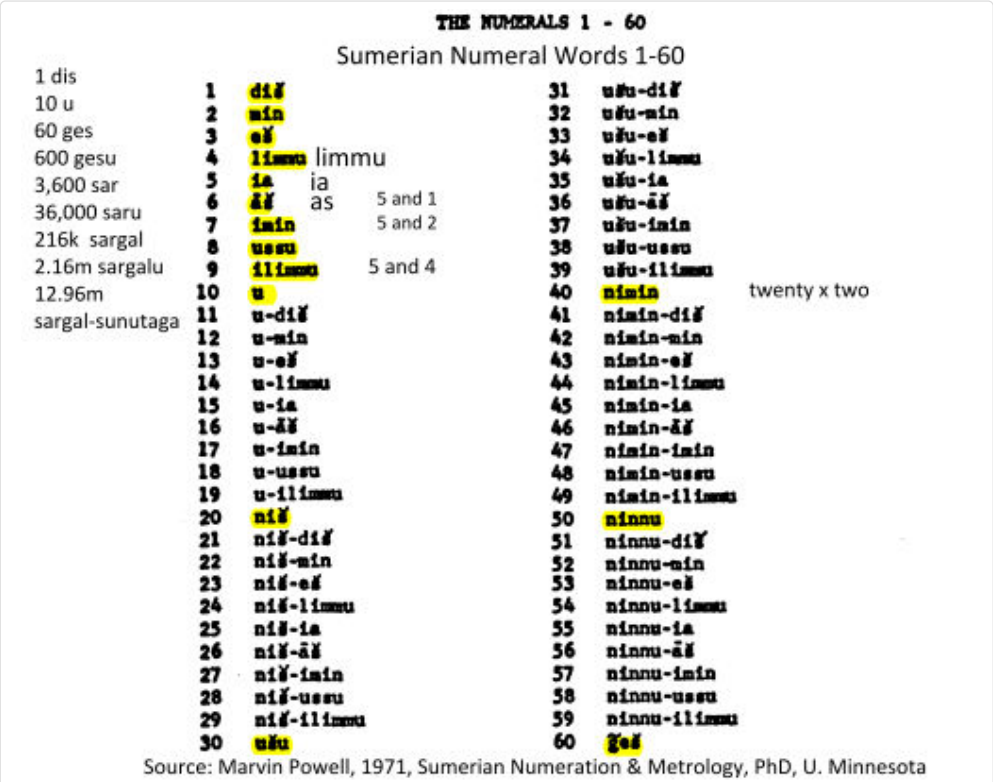
### Sumerian Adjectives – Table 5

**Sumerian Adjectives**

| Dimensions/Time/Weight | | Colors | | Descriptive (related to verbs =a) | |
|---|---|---|---|---|---|
| 1 big | gal | 1 white/light | bar6 | 1 strong | kalag |
| 2 great | mah | 2 black/dark | gi6.g | 2 beautiful | sa6.g |
| 3 small | tur | 3 red/brown | si4 | 3 fierce | hus |
| 4 long | gid2 | 4 yellow | sig17 | 4 big | gul |
| 5 short | lugud | 5 green | sig7 | 5 knowing | zu |
| 6 heavy | dugud | 6 blue | | 6 having | tuku |
| 7 light | | 7 purple | | 7 thick/important | gur4 |
| 8 high | sukud | 8 multi-colored | gun3 | 8 pure | sikil |
| 9 low | | | | 9 far | su.r |
| 10 first, top (head) | sag | | | | |
| 11 last, bottom | ur | **Values** | | **Directions** | |
| 12 old | sumun | 1 pure | ku3.g | 1 right side | zid-a |
| 13 new, young | gibil | 2 right, true | zi.d | 2 left side | gabu2 |
| 14 early, former | libir | 3 false. | lul | | |
| 15 wide | dagal | 4 good, sweet | du10.g | | |
| 16 narrow | sig | 5 bad | hulu | | |
| | | 6 clean | sikil | | |
| **Temperature** | | 7 healthy, complete, whole | silim | | |
| 1 hot | kum | 8 bitter | sis | | |
| 2 cold | sed9 | 9 helpless | hu.nu | | |

Sumerian Adjectives (Source: **Jagersma/2010**, ePSD/2006)

### Sumerian Numerals – Table 6




Sumerian numeral words for the counting system (sexegesimal). Note the alternating x10 and x6 factors, all the way past 12 million. Source: **Powell/1971**, p.47

## Appendix 3. Timelines

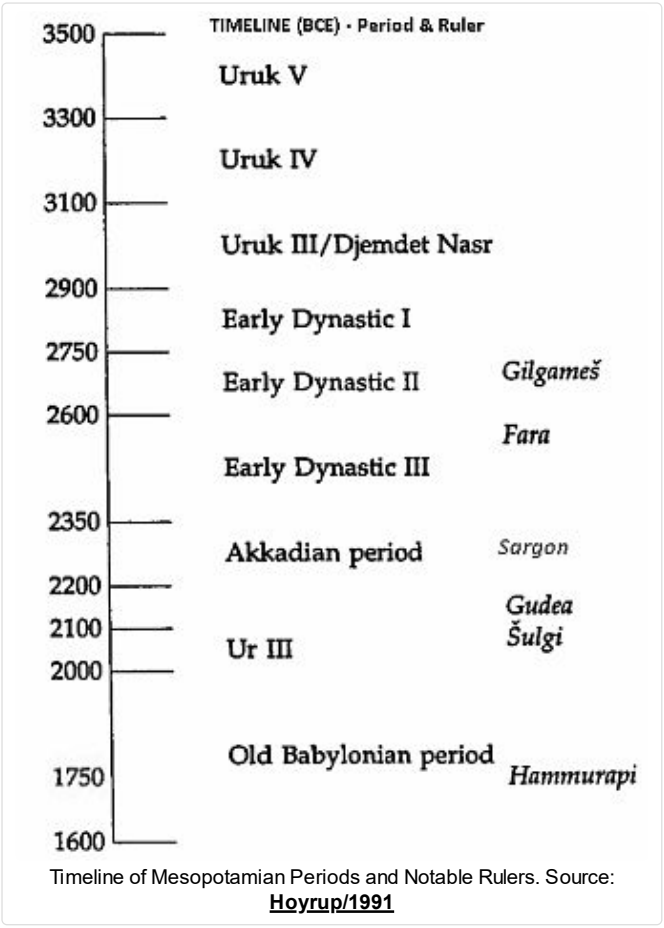
### Simplified Mesopotamian Chronology (Main Periods)

Simplified chronology of ancient Mesopotamia before ca. 2000 BCE.

| | *Period* | *Dating (BCE)* | *Comments* |
|---|---|---|---|
| 10th mill to 6th mill | | 10,000 - 5300 | Neolithic period in Near East |
| 6th/5th mill. | Hassuna / Samarra / Halaf | ~5500–4500 | Neolithic / Chalcolithic archaeological cultures |
| 5th/4th mill. | Ubaid | ~5000–3600 | rural settlements in southern Mesopotamia; introduction of artificial irrigation |
| 4th mill. | Uruk (Uruk V/IV) | ~3600–3100 | large-scale urbanisation; first pictographic script |
| 4th/3rd mill. | Jemdet Nasr (Uruk III) | ~3100–2900 | development of the cuneiform script |
| 3rd mill. | Early Dynastic I/III | ~2900–2350 | many Sumerian cities-states in southern Mesopotamia; Semitic states in the north |
| Late 3rd mill. | Akkadian/Gutean | ~2350–2100 | unification of Mesopotamia by a Semitic dynasty; invasion of Guteans from Gutium |
| End of 3rd mill. | Ur III | ~2100–2000 | re-unification by a Sumerian dynasty |

Simplified Timeline of Mesopotamian Chronology, Source: **Soltysiak/2004**

### Mesopotamian Chronology (Key Periods and Notable Rulers)




Timeline of Mesopotamian Periods and Notable Rulers. Source: **Hoyrup/1991**

### Development of Mesopotamian Mathematics (4000 BCE onwards)

TABLE 2.2: Overview of Mathematical Developments in, Ancient Mesopotamia

| *Date* | *Mathematical developments* | *Socio-political background*[11] |
|---|---|---|
| 4000 BCE | Pre-3200: Preliterate token-based accounting | Increasing urbanization in southern Iraq |
| 3500 BCE | 3200: Literate numeracy; the first school mathematics | Uruk period/Early Bronze Age<br>Sumerian language |
| 3000 BCE | Sophisticated accounting and quantitative planning | Early Dynastic period: city states |
| 2500 BCE | School mathematics; c. 2050: first attestation of the mature sexagismal place value system | Akkadian language<br>Territorial empires of Akkad and Ur |
| 2000 BCE | c. 1850–1650: widespread evidence of "pure" mathematics in scribal training: line geometry, concrete algebra, quantity surveying | City states; empire of Babylon: Middle Bronze Age, or Old Babylonian period |
| 1500 BCE | Cuneiform culture and sexagesimal numeracy spread from southern Iraq | "Amarna age" of international diplomatic contact across the Middle East; Late Bronze Age |
| 1000 BCE | 800 BCE–: quantitive methods in Assyrian scholarship | Assyrian empire; Aramaic language and the alphabet; Iron Age |
| 500 BCE | 400 BCE–: mathematics in the temples of Uruk and Babylon | Persian and Seleucid empires: Late Babylonian period |
| 0 BCE/CE | 75 CE: the last known datable cuneiform tablet; transmission of mathematical knowledge and practice to other languages | Parthian empire |

Development of Mesopotamian Mathematics, 4000 BCE onwards. Source: **Robson/2007**

## Appendix 4. Primary Sources and Research Aids

Where does one find the primary sources and research aids?

1. **CDLI (Cuneiform Digital Library Initiative)**, a joint project of UCLA, University of Oxford, and the Max Planck Institute for the History of Science (Berlin), aims to store digitally high resolution images, line drawings, and transliterations of all known cuneiform texts. It is a fantastic resource! You can put any of the tablet names in the publication box, or specify one of approx 100 search attributes in Full Search.


**Cuneiform Digital Library Initiative (CDLI)** aims to store digitally high resolution images, line drawings, and transliterations of all known cuneiform texts.

2. For proto-cuneiform, you will need access to the latest **archaic sign list**, hosted by CDLI.
3. What do the signs mean? You need **sign readings list**, hosted by CDLI.
4. **Pre-Uruk (8500-3500 BCE) and Uruk V (3500-3350 BCE) periods: 632 texts**.
5. **Uruk IV (3350-3200 BCE) period: 1861 texts**
6. **Cornell's Cuneiform Library** with 219 texts from their archaic collection
7. **Sumerian/Akkadian and English dictionary**, hosted by Penn State.
8. **DCCLT (Digital Corpus of Cuneiform Lexical Texts)**, e.g. **Lexical List LU2 A (standard professions list)**, from ORACC, with links to the attested tables. Example:
   LU2 A Lexical List of Standard Professions, from 3200 BCE (Uruk IV) through to the Fara schooltexts.
   Source: **Englund/1998**, p.104, Fig 32.
   Transliteration: **ORACC**
   Tablet attestation: **MS 2429** (from Umma, c.3200-3000 Uruk III period)
9. Old books (with expired copyrights) from **Archive.org**, e.g. The Sumerian Kinglist by Thorkild Jacobsen
10. **Abbreviations for Assyriology**
11. Academia.edu, a central hub for papers on Assyriology by leading researchers, e.g. **Jens Hoyrup's papers** (x238), **Eleanor Robson's papers** (x81), **Joran Friberg's papers**, **Christine Proust's papers**
12. Homepages of key researchers with their publications: **Robert Englund's publications at CDLI**, **Joran Friberg's publications at Chalmers U.** and **his staff page**, **Jens Hoyrup's page**, **Hans**

**Nissen's page**

13. Google Scholar for citations, cross-references, and PDF papers online, e.g. **Robert Englund's work**
14. **Proto-cuneiform short history and bibliography** (on CDLI)
15. **Intro to Sumerian language and culture, primary sources collection** from course at Masaryk University (Czech). Other courses: **Art and iconography**, **Neolithic Pottery of Near East**, **The Chalcolithic Near East**, **Course 49**
16. **Christie's auction of the Erlenmeyer Collection** (most of which was bought by the Government of Berlin)
17. **ORACC (Open Richly Annotated Cuneiform Collection)** and **List of dozens of collaborative projects**
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19. **DCCMT (Digital Corpus of Cuneiform Mathematical Texts)**, by Eleanor Robson of Oxford University
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21. **MSVO 1, 241 tablets**
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