The Prehistoric Origins of Mathematics

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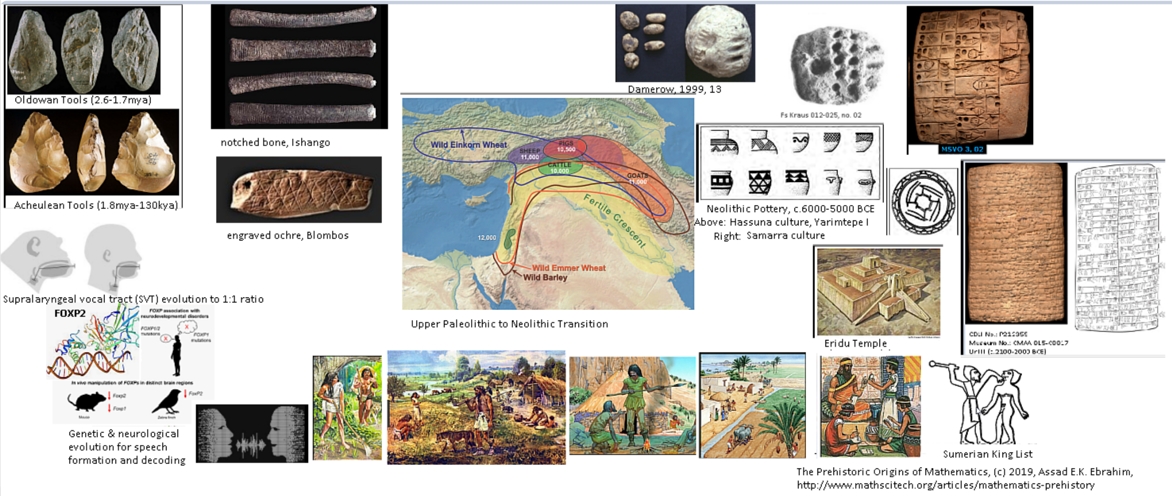
*Part 1 in Ancient Mathematics series. (**Part 2: The Mathematics of Uruk and Susa 3500-3000 BCE**, **Part 3: Exploring Cuneiform Culture 8500-2500 BCE**)*

Abstract

How far back in time can we trace mathematical understanding and mathematical practice? When did we acquire the neurological circuitry for the sense perceptions and cognitive and linguistic capabilities on which mathematics depends? Over the past 50 years (and especially the past 30), there have been an array of advances that have fundamentally changed what we know about our past and about the biological capacity for and cultural impulses behind cognitive precision (language, number sense, cultural transmission). These questions will take us on a journey across multiple disciplines including archaeology, Assyriology, artifact analysis (close reading theory), anthropology, genomics, linguistics, neurobiology, and animal cognition.



The Anthropology and Archaeology of Conceptual Thought leading to the Birth of Mathematics

SUMMARY

In this article, we will proceed backwards in time, starting from c.3000 BCE just after the the birth of written mathematics in Sumeria (modern S.Iraq). From here we will move to the prehistoric evidence of practitioner geometry in the cultures of the late Neolithic as evidenced in the layout of permanent houses, granaries and temples (c.5000 BCE in Ubaid culture), and in geometric pottery designs (c.6000 BCE, Halaf and Samarran cultures). Further back, we see the appearance of plain tokens from c.7000-8000 BCE, the same plain tokens that we know were used for counting by herders and which were instrumental in the invention of writing for book-keeping purposes within the temple institutions running centralized economic control in the urban city-states. Looking beyond the Near East, in Paeleolithic/Neolithic Europe and Britain there is evidence for monolithic monuments c.4000-7000 BCE oriented toward midwinter and midsummer solstice that suggest an awareness of the periodicity of the solar and lunar cycles, and the relation of the solar cycle to the seasons.

Before 10,000 BCE, before the retreat of the last ice age, the density of artifactual evidence is insufficient to draw firm conclusions: there are fewer than 5 isolated finds of artifacts between 70,000-18,000 BCE (one find per 10,000 years), with contested interpretations. So we switch to indirect evidence (genetic, anthropologic, and linguistic) to establish the capability of symbolic thought in **anatomically modern humans (H. Sapiens)** from c.200,000 BCE onwards. Here culture becomes a critical factor, under-scored by the fascinating example of the modern Piraha tribe in Brazil that have broken previous assumptions about the inevitability of symbolic thought in anatomically modern humans. The Piraha are the only known tribe/people whose language and culture appear not to have progressed beyond an analog notion of magnitude similar to that of higher animals, skipping entirely the granular linguistic numeracy present in every other known language, primitive or modern. Why? It appears to be cultural: the Piraha reject the value of future planning and are completely non-materialistic. This leads to an interesting philosophical observation: quantitative mathematics initially develops within a culture that values planning and material control.

From anatomically modern humans c.200,000, we take a big jump backwards to c.2.5 million years ago and consider the capacity for conceptual thinking implicit in the tool-making capability of early hominids. We look at C.S. Peirce's **"semiotic model" (index, icon, symbol)** of conceptual and linguistic development, and conclude that bladed tool-making required at least stage 2 or stage 3 conceptual development. Having gone back as far as we can with the capabilities of humans and hominids, we consider the origin of number sense in humans, animals, birds, and reptiles, and trace back the neurological circuitry supporting an analogue number sense to a latest common ancestor (LCA), a stem reptile that would have existed some 260 million years ago.

A set of **Appendices** provide additional color on:

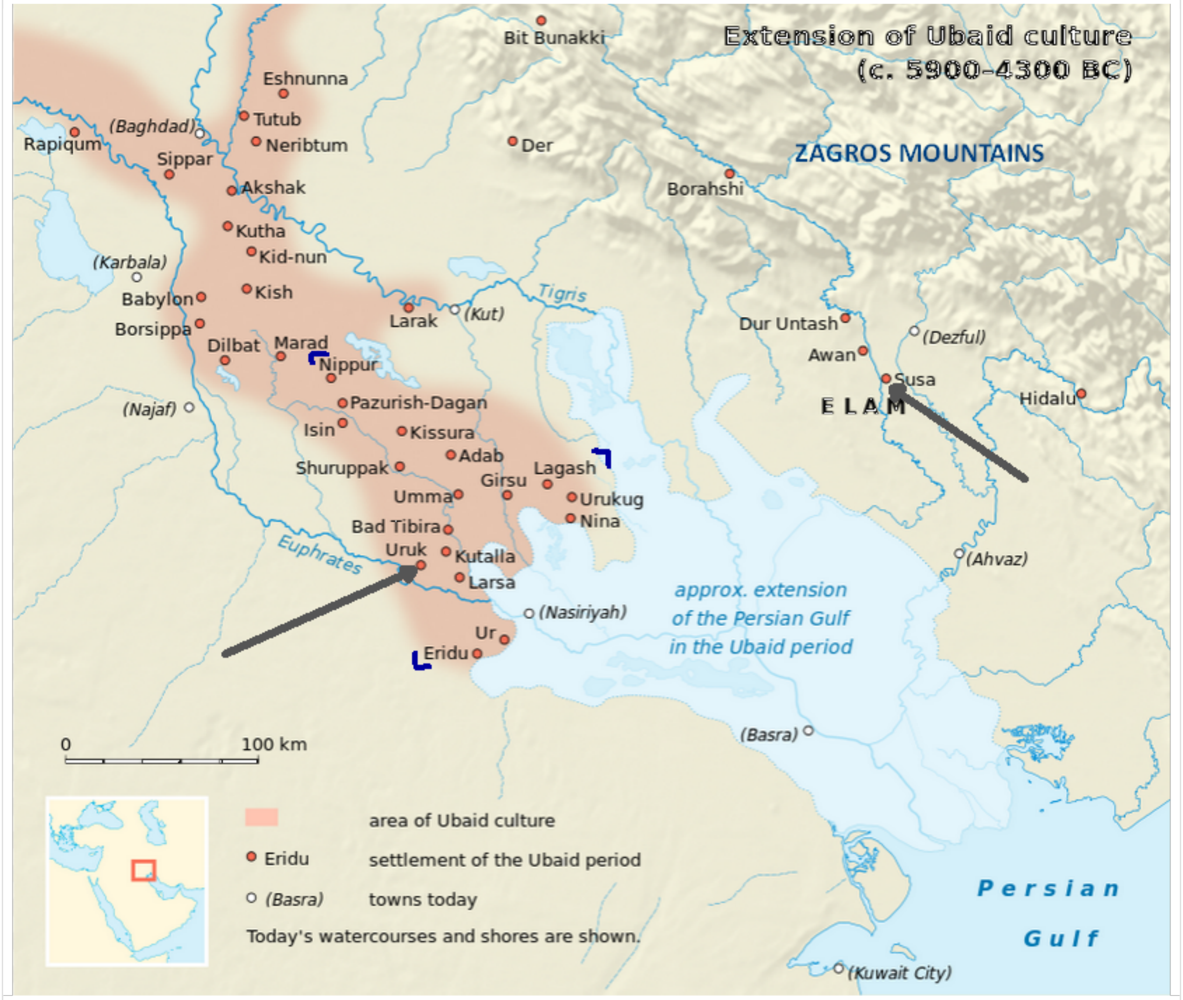
1. **Appendix 1**: the dialectic nature of arithmetic (and mathematics),
2. **Appendix 2**: the invention of writing and the advancement of book-keeping
3. **Appendix 3**: Near Eastern Cultural History: from Shanidar to Uruk
4. **Appendix 4**: timeline of the stone age
5. **Appendix 5**: what life was like in the transition to the neolithic period
6. **Appendix 6**: timeline for the domestication of animals.
7. **Appendix 7**: the evolution of humanity
8. **Appendix 8**: birth of the universe to the origin of man

A list of recommended readings, most of which can be downloaded freely, is provided in the **Bibliography**.

1. Evidence from the dawn of written mathematics (c.3000 BCE): accounting with clay tokens.

By 3,200 BCE (5200 years ago) there is indubitable evidence for mathematical practice within the sophisticated cultural context of Neolithic Sumerian city states with a strong centralized control of production resources and economic activity through temple-statal administration. This led to the breakthrough advancement of proto-writing: scribes used the clay tokens typically kept in "bullae" (clay envelopes) to impress upon flattened clay tablets to create the earliest known system of accounting, or book-keeping. In this context, the token combined quantity and commodity, so correct interpretation required knowing the context of the transaction. Within 50 years, there was a further advance: pictographic signs that could specify commodity separately from quantity. Over the next 500 years, temple and state control of economic planning and supply chain management grew more extensive and more ambitious, developing syllabic writing, standardizing traditional metrologies (the same signs could still take on different values depending on metrological context), and improving arithmetic technologies (the emergence of the sexagesimal system, reciprocal tables, other aids to calculation/computation/solving problems).

The strongest archaeological evidence of mathematical practice dates to at least 3,200 BCE (5200 years ago) in ancient near eastern city-states. Archaeological finds in the past century have shown that geometrical clay tokens which appear to have been used for counting and measuring across the region, became established at this time in the Sumerian city-state of Uruk (southern Mesopotamia/Iraq) as the standard administrative procedure for recording commercial transactions (archaic book-keeping). Similar finds have been made in Elamite Susa (Zagros mountains/western Iran), a rival city-state to Uruk. [**NissenDE/1993**], [**Friberg/1984**].



Near East toward the end of the Ubaid period (4300 BCE), before the earliest discovery of accounting. Notice the many city-states that had arisen in the alluvial flood plains between the Tigris and Euphrates rivers. Uruk and Susa would rise as leading city-states in Sumer and Elam respectively. The blue markers identify what would be considered the three religious centres in the following millenia. (Source: **Wikipedia**)

Key to this conclusion were the finds by Denise Schmandt-Besserat of tokens enclosed in clay "bullae", or sealed clay envelopes, with matching token-impressions on the clay surface. ([**Besserat/1977**], [**Oppenheim/1959**]) The impressed indentations made by pushing the tokens into the wet clay are also the earliest examples of proto-writing. [**Damerow/1999w**]

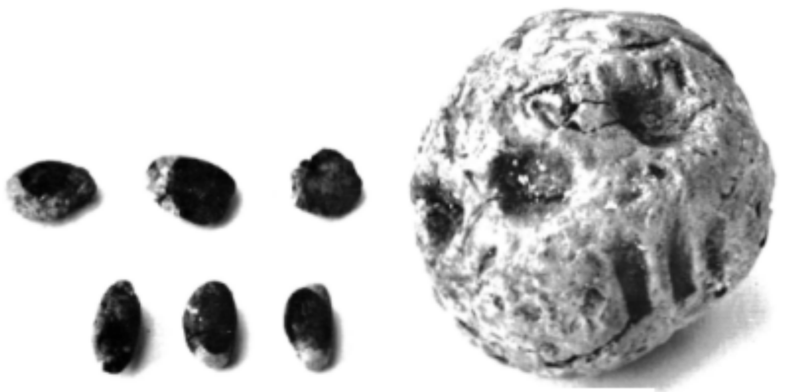
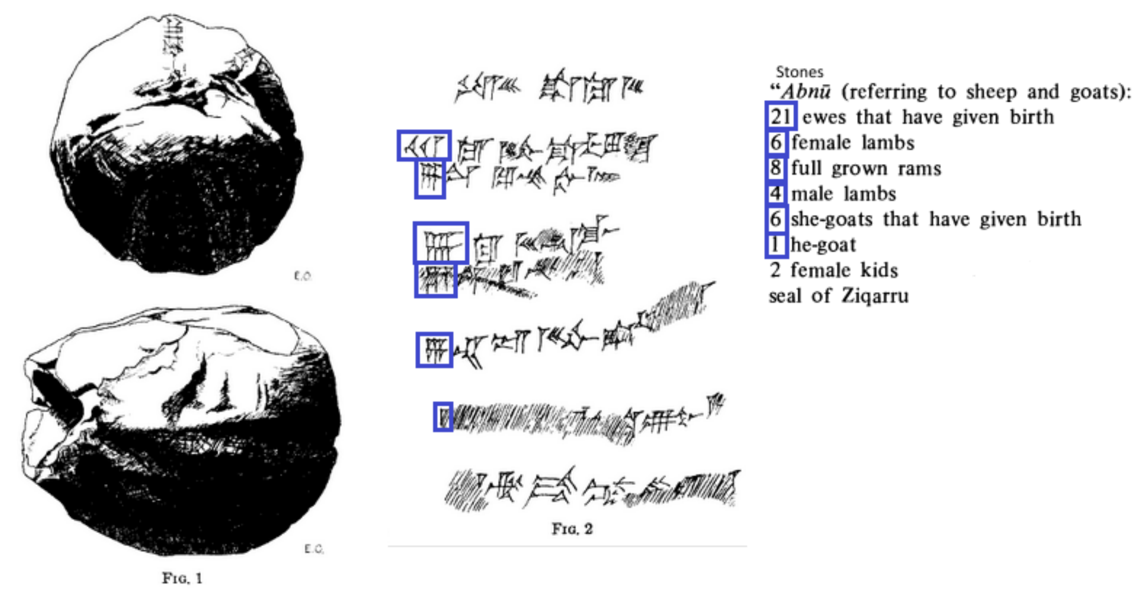
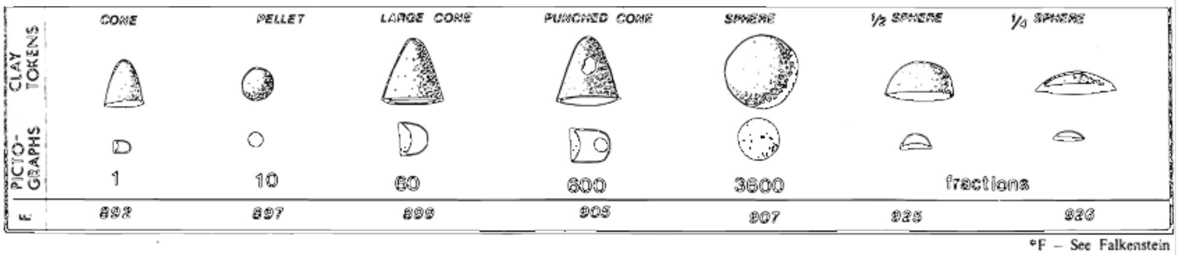


Figure 8. Susa, Iran. Bulla with its contents:
3 discs and 3 elongated pellets.
Note the corresponding marks on the outside of the bulla.

Clay tokens and bulla (envelope). Note the impressions of the tokens on the surface of the bulla (Source: **Besserat/1977**, SMS 1, p53)



Bulla with 48 tokens found at Nuzi site dated from 2000-1500 BCE after cuneiform writing was fully developed, with an inscription describing the meaning of the tokens inside. (Source: **Oppenheim/1959**)

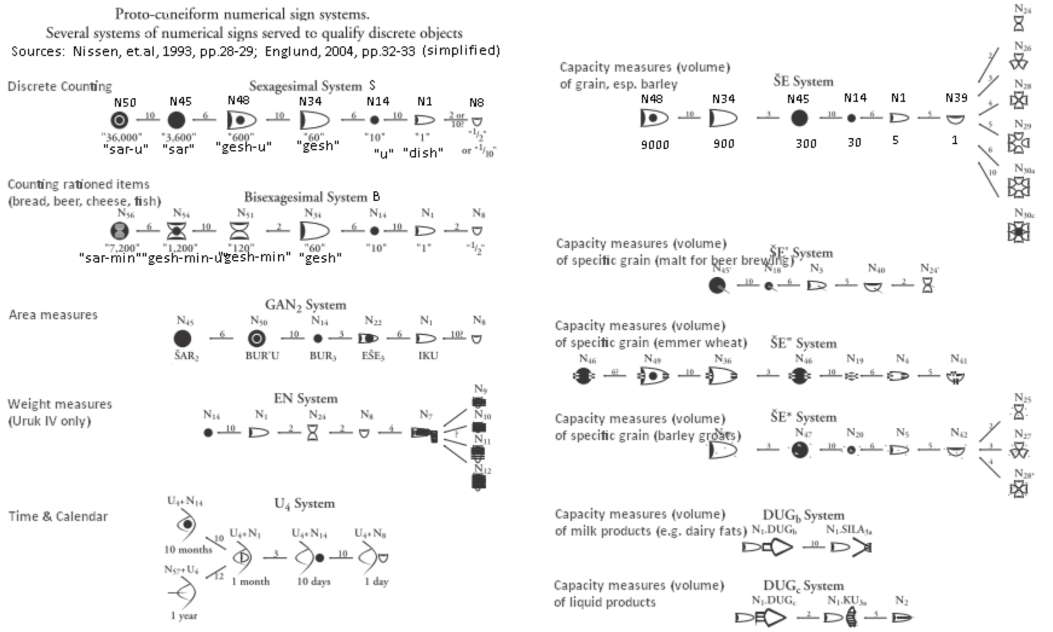


Clay tokens mapped to the pictographs and numerical values assigned to them. (Source: **Besserat/1977**]

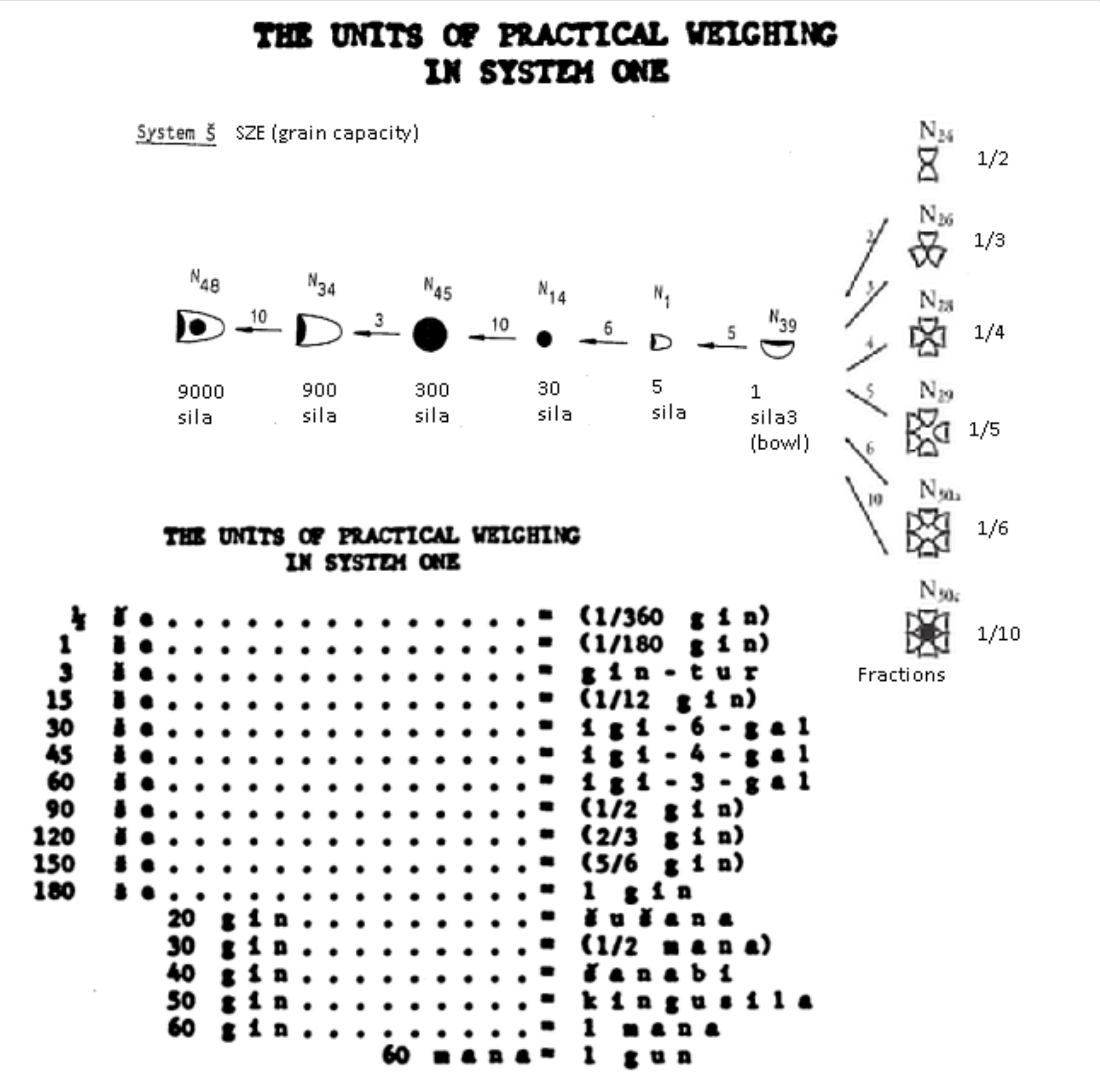
The meaning of these impressions can be worked out from the historical sequence of clay tablets that initially record token-impressions only with no additional written context, to later the juxtaposition of number signs with additional signs indicating the commodity (e.g. 3 sheep), and finally the use of separate **cuneiform** signs for number and for commodity. [**Nissen/1986**], [**NissenDE/1993**], [**Robson/2000**].

There is evidence of elaborate systems of metrology (measurement) that linked the tokens variously to different length, area, volume, weight, and time units, in nested factors of 2,3,6, and 10. The decipherment of these metrologies was based on painstaking studies of hundreds of archaic tablets with numbers matched to those in cuneiform tablets using the associated cuneiform symbols (see below on the number system). [Powell/1971], [Nissen/1986], [Nissen/1993], [Englund/2004] The situation with ancient Sumerian metrologies was similar to
customary measures in medieval Europe, see also **Mathematics of Uruk and Susa**)

What must be remembered is that the mathematical and metrological understanding that is captured in the earliest tablets in 3200 BCE was *pre-existing* and hence pre-dated 3200 BCE, before writing. It was the technology of book-keeping through writing that was the invention of the time (see the next section).



Early metrology (counting & measurement) used separate systems depending on the commodity being measured. (Source: **Nissen/1993**, pp.28-29, **Englund/2004**, pp.32-33)



THE UNITS OF PRACTICAL WEIGHING
IN SYSTEM ONE

½	še	(1/360 gin)
1	še	(1/180 gin)
3	še	gin-tur
15	še	(1/12 gin)
30	še	igi-6-gal
45	še	igi-4-gal
60	še	igi-3-gal
90	še	(1/2 gin)
120	še	(2/3 gin)
150	še	(5/6 gin)
180	še	1 gin
20	gin	šušana
30	gin	(1/2 mana)
40	gin	šanabi
50	gin	kingusila
60	gin	1 mana
60	mana	1 gun

SZE system for measuring grain capacity in units of 1 sila3 (bowl). Notice the fractions measured in shekels with 180 shekels in a gin, and 360 1/2 shekels.

How far back did the use of plain tokens for counting and measurement go?
We have clear evidence for their use c.3200 BCE for administrative purposes associated with temple management of Ubaid period economy controlling surpluses and labor. Before 3,200 BCE, while plain tokens are find in many sites, they are without sufficient context (e.g. the bullae with imprints) to conclude definitively that they were used for counting and measurement. Thus we can provide only a date range for the start of plain token use for counting and measurement, from 8,000 BCE to 3,200 BCE (see [**Niemi/2016**: 33-34], and [Bennison/2018: 20-22]).

Archaic Tablet Texts

From 3,200 BCE onwards, there is increasing archaeological record of clay tablets [**Friberg/1984**]. The anthropological and sociological work of Nissen, Damerow, Englund, Hoyrup, Robson, and several others, have led since the 1980s to an understanding of how the temple economy evolved the scribal-statal system built around written accounting practice [**Hoyrup/1991**]. Mathematically, this proceeded over 1000 years (i.e. from 3200 BCE to 2300 BCE) in various stages: (1) an initial stage in which quantity and commodity were combined in a single token/impression; (2) adding pictographic symbols for representing the commodity for which the number provided the quantity; (3) logographic cuneiform (writing with the wedge-end of a reed) in which the symbol represented the full word or idea, giving no indication of the pronounciation; (4) syllabic representation of spoken language using the same cuneiform symbols enabling Sumerian and Akkadian scribes to record concepts and literary ideas as well as numerical transactions. [**Nissen/1986**], [**Robson/2008**]

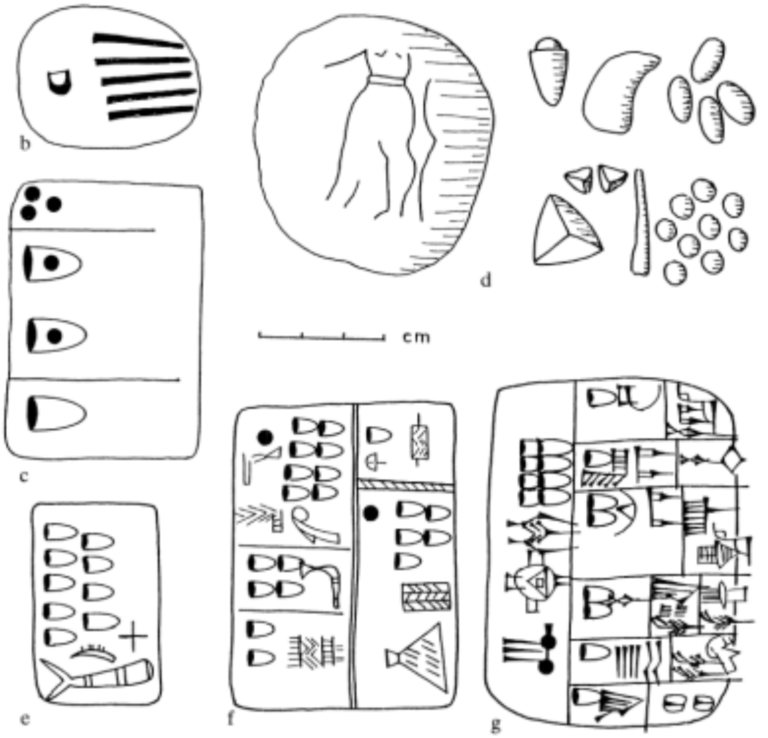
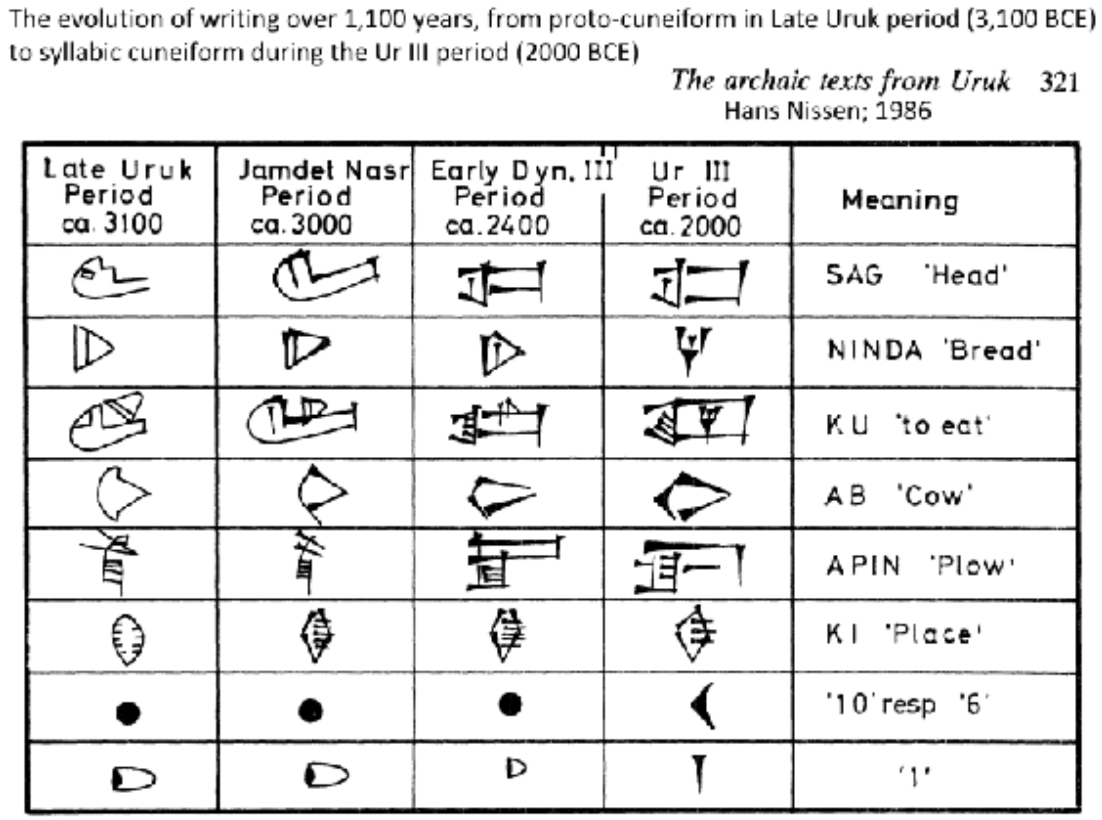


Figure 4 Early controlling devices: (a) Cylinder seal and sealing; (b) simple tablet containing numerical signs only; (c) complex numerical tablet divided into several compartments; (d) clay 'envelope' and clay tokens; (e) simple economic tablet of stage IV; (f) complex economic tablet of stage IV; (g) complex economic tablet of stage III 2.

The complexity of administrative tablets and notation evolved from quantity/commodity combined (commodity implicit) to quantity and commodity indicated with separate signs. (Source: **Nissen/1986**)



The evolution of writing over 1,100 years, from proto-cuneiform in Late Uruk period (3,100 BCE) to syllabic cuneiform during the Ur III period (2000 BCE)

The archaic texts from Uruk 321
Hans Nissen; 1986

Late Uruk Period ca. 3100	Jamdet Nasr Period ca. 3000	Early Dyn. III Period ca. 2400	Ur III Period ca. 2000	Meaning
				SAG 'Head'
				NINDA 'Bread'
				KU 'to eat'
				AB 'Cow'
				APIN 'Plow'
				KI 'Place'
				'10' resp '6'
				'1'

The evolution of writing from pictographs/logographs to cuneiform, as the type of stylus changed. (Source: **Nissen/1986**)

Style of writing evolved from pictographic to cuneiform as the stylus changed. (Source: **Nissen/1986**)

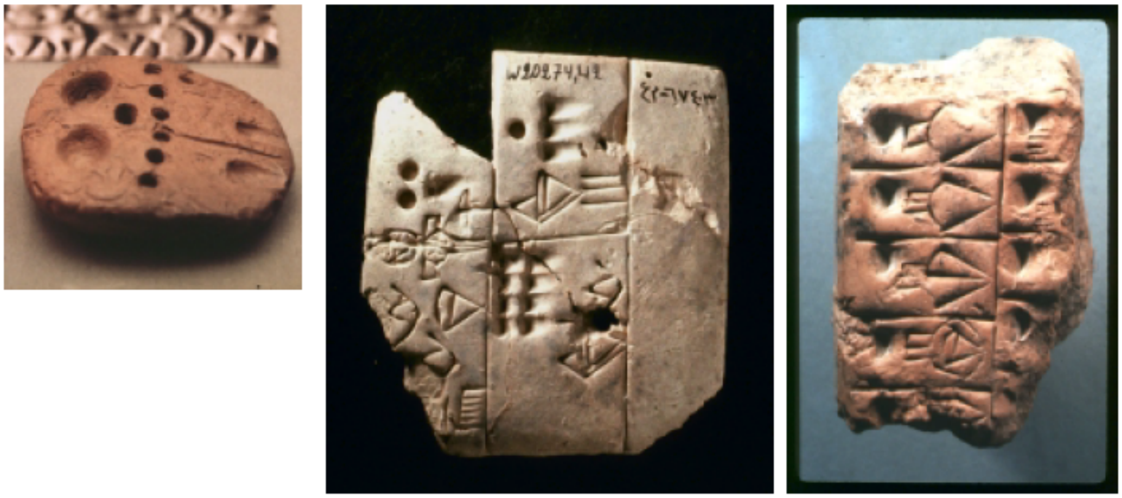
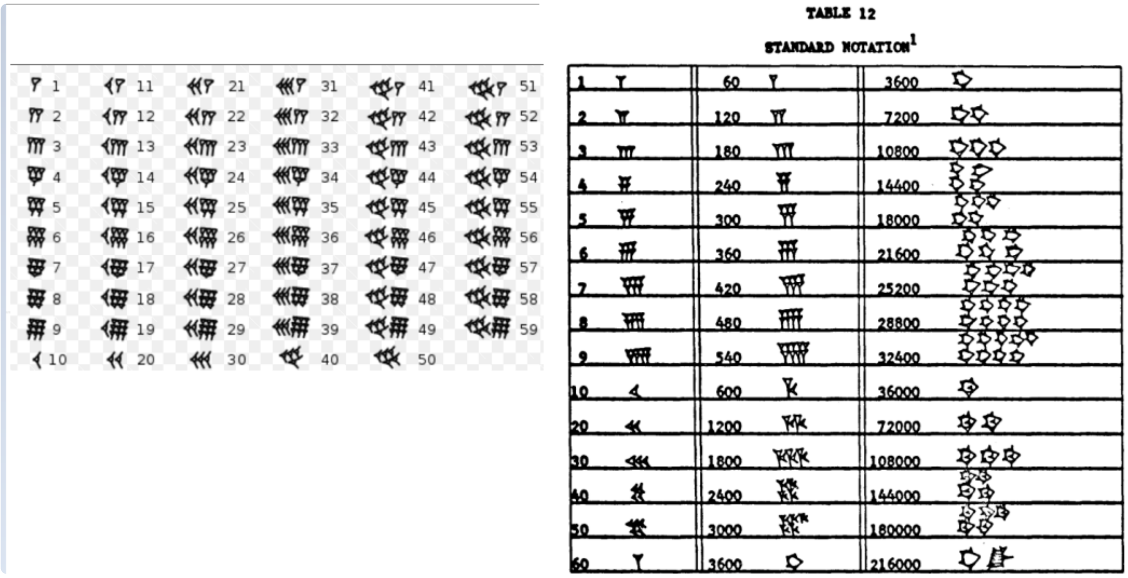


Figure 2. Hierarchically structured proto-cuneiform administrative document (W 20274,42 with an entry and two subentries) and „list" of vessels in a school text (fragment W 24157 of the vessels list, see Englund and Nissen, 1993). Both texts are depicted in conventional orientation turned 90° to the left.

(left) simple clay tablet with token impressions for quantity (commodity is presumably implicitly known by context). The shape of the token impressions led to the early metrological signs. (middle & right) complex numerical tablets showing quantity and commodity separately. (Source: **Damerow/1999w**)

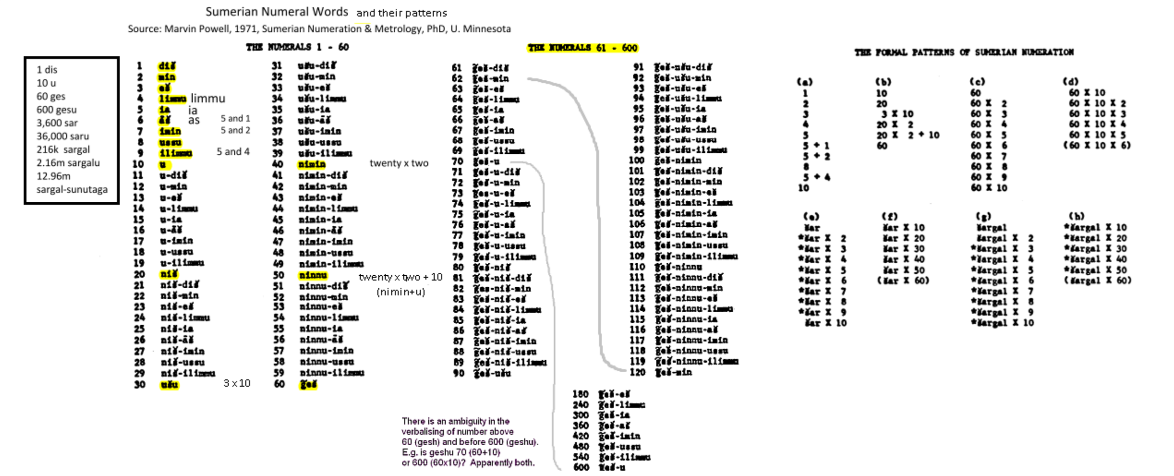
Sumerian written and spoken numerals

The cuneiform representation of the Sumerian/Akkadian/Babylonian number system was rather cumbersome. They had a symbol for 1 (vertical wedge), a symbol for 10 (horizontal wedge nail end only), and a symbol for 60 (the same symbol as for 1, interpreted by context or by position). The digits 1 to 9 were expressed by writing the requisite number of 1's, either consecutively or bundled together in groups of three with one group on top of another. Similarly the 'digits' 10, 20, …, 50 were expressed by the requisite number of 10's, etc. Thus the number 147 would be represented by two symbols of 60, two of 10, and seven of 1. The digit 0 was not used and presumed to be understood from the context, although in the later Babylonian period it was denoted by a wedge. [Roy, 2003] This can be seen:



Cuneiform representation of Sumerian numbers

The spoken numbers show a fascinating linguistic pattern, with some trace of base 5 (the numbers 6 through 9 are named as 5+1,…,5+4, though not consistently), strongly base 10 structure (the numbers 11-19 are 10+1,…,10+9), some trace of base 20 (40 is nimin, or 2×20), and mixing (50 is 2×20+10). Finally the sexagesimal unit 60 is reached (ges) and the pattern repeats.
There is an ambiguity in the verbalizing of numbers higher than 60 (gesh). Is gesh-u 70 (=60+10) or 600 (=60×10)? Apparently both, with the amount resolved in context. Compare the linguistc structure of languages from Inuit (base 20), western languages (nominally base 10 but with various inconsistencies in formulation) [Gullberg, 1997, 7-60], and east Asian languages (base 10 with clean structure). [Takasughi, 1996]



Sumerian Numeral Words and their patterns
Source: Marvin Powell, 1971, Sumerian Numeration & Metrology, PhD, U. Minnesota

1 dis
10 u
60 ges
600 gesu
3,600 sar
36,000 saru
216k sargal
2.16m sargalu
12.96m
sargal-sunutaga

THE NUMERALS 1 - 60

1	diš		31	ušu-diš	
2	min		32	ušu-min	
3	eš		33	ušu-eš	
4	limmu	limmu	34	ušu-limmu	
5	ia	ia	35	ušu-ia	
6	àš	as 5 and 1	36	ušu-àš	
7	imin	5 and 2	37	ušu-imin	
8	ussu		38	ušu-ussu	
9	ilimmu	5 and 4	39	ušu-ilimmu	
10	u		40	nimin	twenty x two
11	u-diš		41	nimin-diš	
12	u-min		42	nimin-min	
13	u-eš		43	nimin-eš	
14	u-limmu		44	nimin-limmu	
15	u-ia		45	nimin-ia	
16	u-àš		46	nimin-àš	
17	u-imin		47	nimin-imin	
18	u-ussu		48	nimin-ussu	
19	u-ilimmu		49	nimin-ilimmu	
20	niš		50	ninnu	twenty x two + 10
21	niš-diš		51	ninnu-diš	(nimin+u)
22	niš-min		52	ninnu-min	
23	niš-eš		53	ninnu-eš	
24	niš-limmu		54	ninnu-limmu	
25	niš-ia		55	ninnu-ia	
26	niš-àš		56	ninnu-àš	
27	niš-imin		57	ninnu-imin	
28	niš-ussu		58	ninnu-ussu	
29	niš-ilimmu		59	ninnu-ilimmu	
30	ušu	3 x 10	60	ĝeš	

THE NUMERALS 61 - 600

61	ĝeš-diš	91	ĝeš-ušu-diš
62	ĝeš-min	92	ĝeš-ušu-min
63	ĝeš-eš	93	ĝeš-ušu-eš
64	ĝeš-limmu	94	ĝeš-ušu-limmu
65	ĝeš-ia	95	ĝeš-ušu-ia
66	ĝeš-aš	96	ĝeš-ušu-aš
67	ĝeš-imin	97	ĝeš-ušu-imin
68	ĝeš-ussu	98	ĝeš-ušu-ussu
69	ĝeš-ilimmu	99	ĝeš-ušu-ilimmu
70	ĝeš-u	100	ĝeš-nimin
71	ĝeš-u-diš	101	ĝeš-nimin-diš
72	ĝeš-u-min	102	ĝeš-nimin-min
73	ĝes-u-eš	103	ĝeš-nimin-eš
74	ĝeš-u-limmu	104	ĝeš-nimin-limmu
75	ĝeš-u-ia	105	ĝeš-nimin-ia
76	ĝeš-u-aš	106	ĝeš-nimin-aš
77	ĝeš-u-imin	107	ĝeš-nimin-imin
78	ĝeš-u-ussu	108	ĝeš-nimin-ussu
79	ĝeš-u-ilimmu	109	ĝeš-nimin-ilimmu
80	ĝeš-niš	110	ĝeš-ninnu
81	ĝeš-niš-diš	111	ĝeš-ninnu-diš
82	ĝes-niš-min	112	ĝeš-ninnu-min
83	ĝeš-niš-eš	113	ĝeš-ninnu-eš
84	ĝeš-niš-limmu	114	ĝeš-ninnu-limmu
85	ĝeš-niš-ia	115	ĝeš-ninnu-ia
86	ĝeš-niš-aš	116	ĝeš-ninnu-aš
87	ĝeš-niš-imin	117	ĝeš-ninnu-imin
88	ĝeš-niš-ussu	118	ĝeš-ninnu-ussu
89	ĝeš-niš-ilimmu	119	ĝeš-ninnu-ilimmu
90	ĝeš-ušu	120	ĝeš-min

180	ĝeš-eš
240	ĝeš-limmu
300	ĝeš-ia
360	ĝeš-aš
420	ĝeš-imin
480	ĝeš-ussu
540	ĝeš-ilimmu
600	ĝeš-u

There is an ambiguity in the verbalising of number above 60 (gesh) and before 600 (geshu). E.g. is geshu 70 (60+10) or 600 (60x10)? Apparently both.

THE FORMAL PATTERNS OF SUMERIAN NUMERATION

(a)	(b)	(c)	(d)
1	10	60	60 X 10
2	20	60 X 2	60 X 10 X 2
3	3 X 10	60 X 3	60 X 10 X 3
4	20 X 2	60 X 4	60 X 10 X 4
5	20 X 2 + 10	60 X 5	60 X 10 X 5
5 + 1	60	60 X 6	(60 X 10 X 6)
5 + 2		60 X 7	
8		60 X 8	
5 + 4		60 X 9	
10		60 X 10	

(e)	(f)	(g)	(h)
šar	šar X 10	šargal	*šargal X 10
*šar X 2	šar X 20	šargal X 2	*šargal X 20
*šar X 3	šar X 30	*šargal X 3	*šargal X 30
*šar X 4	šar X 40	*šargal X 4	*šargal X 40
*šar X 5	šar X 50	*šargal X 5	*šargal X 50
*šar X 6	(šar X 60)	*šargal X 6	(šargal X 60)
*šar X 7		*šargal X 7	
*šar X 8		*šargal X 8	
*šar X 9		*šargal X 9	
šar X 10		*šargal X 10	

Sumerian number words and counting patterns

The deciphering of cuneiform languages: Sumerian, Akkadian, Babylonian, and Old Persian

How do we know what these signs mean? Our understanding of cuneiform is relatively recent, with pioneering deciphering work beginning from 1838 (CE) onwards [**Friberg/1984**]. But our understanding of Mesopotamian mathematics is burdened by how we came to this understanding. Before 1850, the major investigators were **Edward Hincks**, Henry Rawlinson, and Jules Oppert, who collectively deciphered the language working off the tri-lingual Behistun Inscription (see section below). From the 1850s onward, the focus was on deciphering specific tablet finds. The first set of tablets discovered pre-1945 were very different from the ones from 1945-1951 (Susa tablets). The investigators of the first set of tablets were either philologists (language studyiers, Abraham Sachs, Goetze) or mathematicians (Neugebauer, Thureau-Dangin) but rarely both. Neugebauer was the closest to both, and he, as well as others, fell victim to using modern symbolism, and interpreting Babylonian discussions in an modern, even pre-modern context. By the time the next set of data had come in, there had been several decades and multiple secondary sources telling the story that Babylonian mathematics was the antecedant of the Greek mathematics, and that Babylon was the origin of the stream that fed the rest of Western Mathematics. Thus, in terms of what is known about Babylonian mathematics, one should disregard all that was thought to be known up to the 80s, and certainly the received knowledge that is in textbooks and histories even through the 2010s. The new knowledge began in the 1970s from a revised look by Marvin Powell and Joran Friberg at existing works, and by Denise Schmandt-Besserat at archaeological findings. All three showed that Mesopotamian mathematics needed to be seen in historical development. [Hoyrup/1991b: 27]. The new knowledge from the 1980s is not only from new sources, corrected transliterations, and better translation/interpretation (e.g. Hoyrup's conformal translations), but also new information about cultural context (scribal roles, social/economic/political/military trends, use of clay tokens, and location of finds — i.e. exercise books, teacher training (e.g. Susa tablets), scrapheap (exams/discards), problem sets with abbreviated solutions for instructors. These insights were contributed by Hans Nissen, Robert Englund, Peter Damerow, Jens Hoyrup, Joran Friberg, and Marvin Powell, who, amongst others, were part of the Berlin Workshop on Concept Development in Babylonian Mathematics. [Hoyrup/1991b]

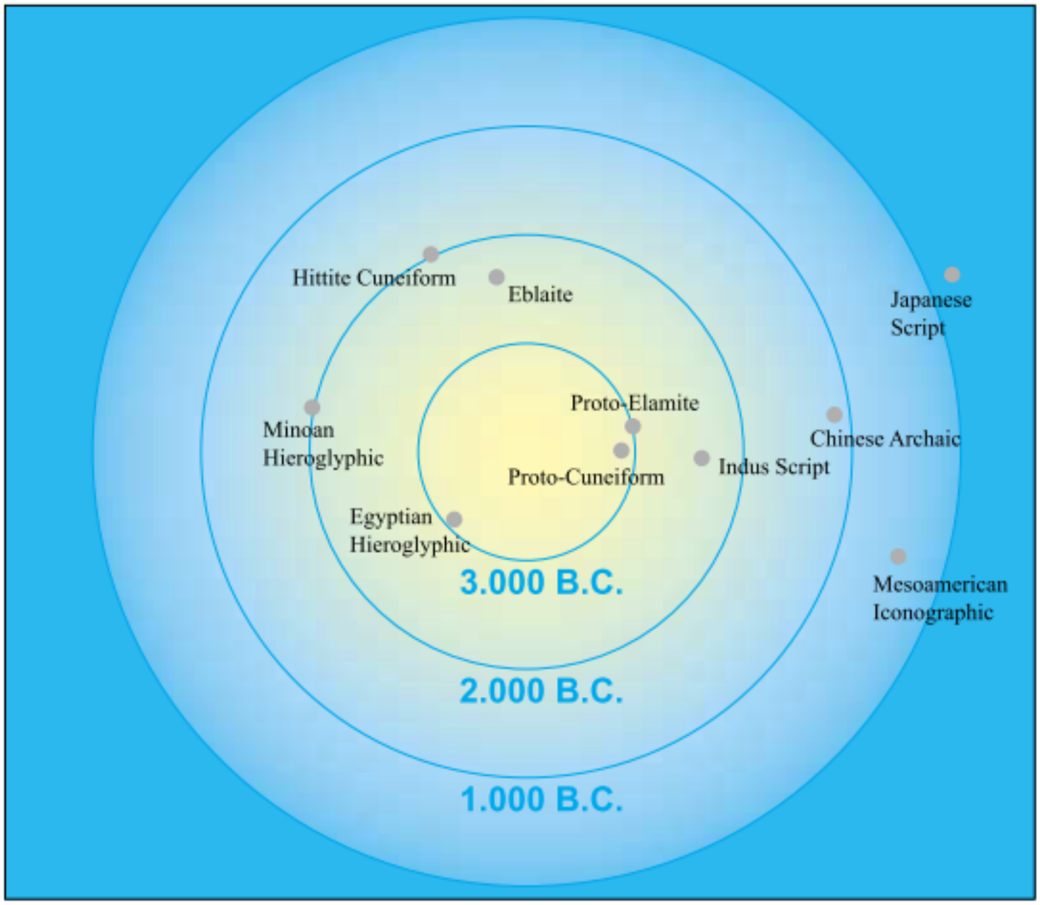


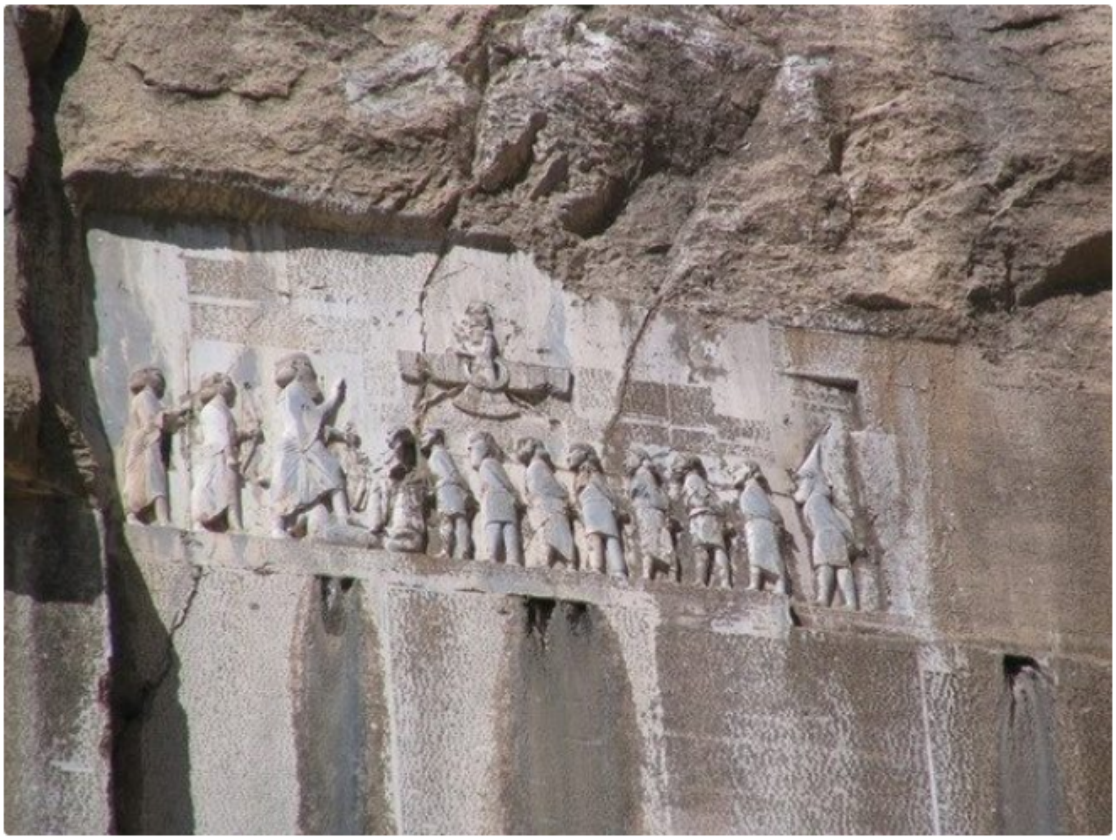
Figure 1. Historio-Geographic Map of Earliest Attestations of Writing

Earliest attestations of writing, beginning with proto-cuneiform c.3200 BCE (Source: **Damerow/1999**)

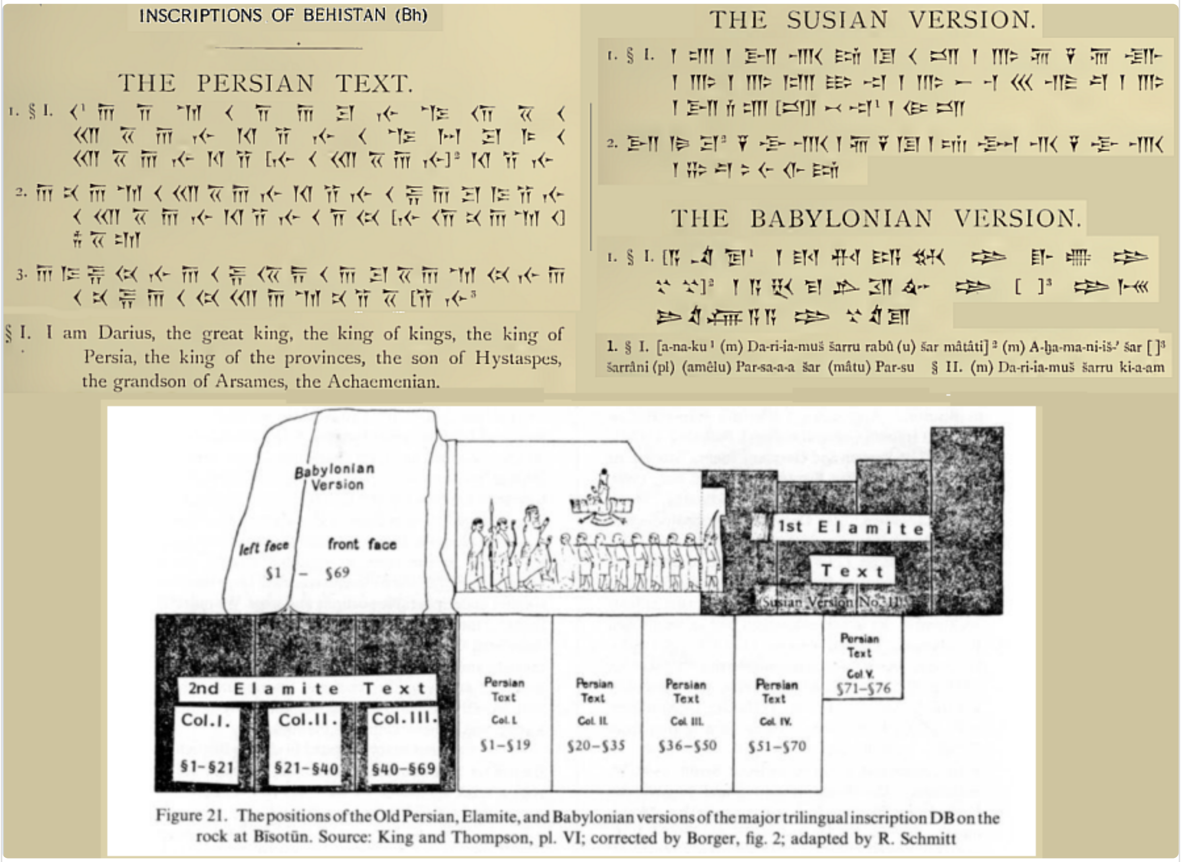
Behistun: from Herodotus to Old Persian to Babylonian Akkadian

The trilingual **Behistun Inscription** is to cuneiform writing what the rosetta stone (discovered 1799) is for the understanding of Egyptian hieroglyphics. The Inscription was engraved into the face of sheer cliffs near the ancient crossroads of Behistun (Kermanshah province of Iran) by the Achaemenid (Persian) king Darius the Great (Darius I) in 550 BCE. Its message proclaiming his conquest of all the lands and his right to rule, was intended for the entire Near East, and so was written in the three major cuneiform languages of his day: Old Persian (his own language), Elamite (the language of Susa), and Babylonian Akkadian (the semetic language understood across Assyria and Mesopotamia). All three languages had died even by 400 BCE and over the millenia fanciful suggestions were put forth as to what the inscription signified. **Additional background on the Behistun Inscription (2017)**

In 1802, Grotefend had deciphered ten of the 37 symbols of Old Persion. Sir Henry Rawlinson started in 1835 using Grotefend's efforts. He found the first part of the Inscription contained the same list of Persian kings as given in Herodotus (400 BCE) but in their Persian forms. By 1838 Rawlinson had succeeded, in part due to the fact that Old Persian used an efficient syllabic representation of 37 characters. In 1844 and 1847 he studied the Babylonian section. Edwin Norris, a colleague, completed the study of the Elamite section by 1855. By 1855 [**Rawlinson/1855**] and Norris with a few others (Hincks 1854) had deciphered all three cuneiform sections: Old Persian (37 characters), Elamite (131 characters) and Babylonian (500 characters, more than 10x the number for syllabic Old Persian). The decipherment of Akkadian and Sumerian (**Cathcart, 2011 paper**)



Trilingual Behistun Inscription carved into sheer cliffs in the Kermanshah province of Iran, engraved c.550 BCE by the Achaemenid king Darius the Great.



INSCRIPTIONS OF BEHISTAN (Bh)

THE PERSIAN TEXT.

§ I. I am Darius, the great king, the king of kings, the king of Persia, the king of the provinces, the son of Hystaspes, the grandson of Arsames, the Achaemenian.

THE SUSIAN VERSION.

THE BABYLONIAN VERSION.

1. § I. [a-na-ku [1] (m) Da-ri-ia-muš šarru rabû (u) šar mâtâti] [2] (m) A-ḫa-ma-ni-iš-' šar [ ][3] šarrâni (pl) (amêlu) Par-sa-a-a šar (mâtu) Par-su § II. (m) Da-ri-ia-muš šarru ki-a-am

Figure 21. The positions of the Old Persian, Elamite, and Babylonian versions of the major trilingual inscription DB on the rock at Bīsotūn. Source: King and Thompson, pl. VI; corrected by Borger, fig. 2; adapted by R. Schmitt

The trilingual inscription at Behistun commissioned by Darius the Great in 550 BCE with messages written in the three cuneiform languages of the time: Old Persian, Elamite, and Babylonian (Source: **LW King & RC Thompson, 1907**

From Old Persian to Babylonian to Akkadian to Sumerian

Behistun provided the key to Babylonian through **Old Persian**, which is accessible through Middle and Modern Persian (Farsi). Babylonian (and Assyrian) Akkadian are derivative dialects of an older Semetic language **Akkadian**. Their decipherment was completed by Hincks, Rawlinson, and Oppert in the mid 1800s, and from 1956 through 2011, the 26-volume Akkadian dictionary was compiled by University of Chicago (freely available online). To get to **Sumerian** we needed to rely on Sumero-Akkadian bilingual texts, and fortunately there are many, primarily the sign lists written by the early scribes that lived during the time of the transition from Sumerian city states to the Akkadian Empire, after Sargon unified the Sumerian city states under his rule. **History of Akkadian (2003)**

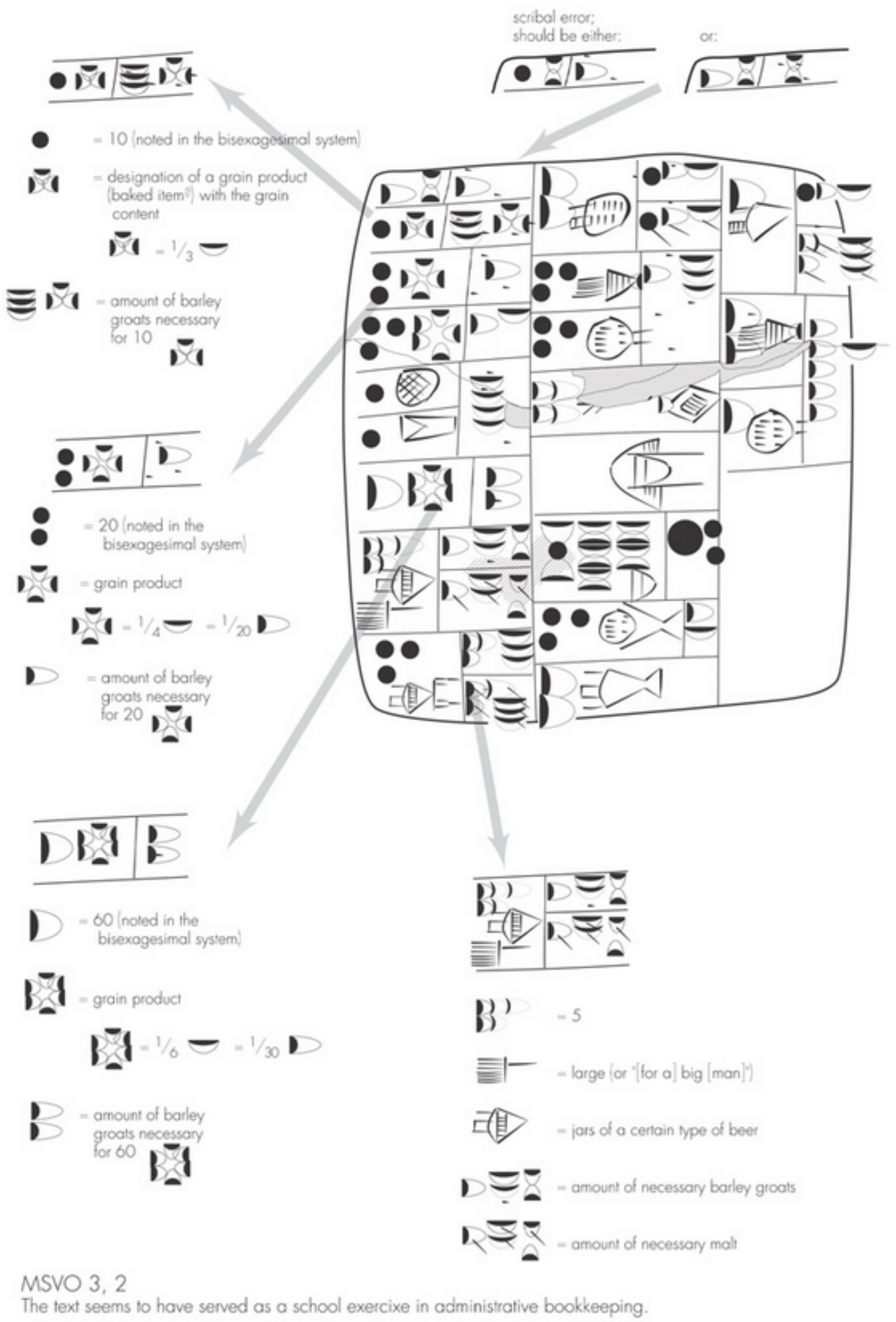
Deciphering proto-cuneiform (pictographs) from Sumerian cuneiform

Deciphering an unknown but syllabic written language is hard. Deciphering the meaning of pictographs is harder still. To get to the meaning of the proto-literate writings it has taken the efforts of the Berlin group, a cross-disciplinary group of researchers who have used computer aided digitization of dozens of fragments to complete the work begun in 1936 by Adam Falkenstein who first published the Archaic Texts of Uruk (ATU).

Nissen 1986 (p.317) explains: even in 1936 it was recognized that a few texts were lists. Later more lists were discovered. And it was noticed that these proto-cuneiform lists corresponded word for word, position for position with the same lists almost 600 years later which were written in Sumerian cuneiform which by this time we did understand.

Figure 3. Bevelled-rim bowl (left) used for the disbursement of rations represented by the sign NINDA (middle, left column) which could be used for designating a ration of a certain size or in a semantically defined sign combination for designating an institution (middle, left column). In combination with a man‘s head it formed the sign combination GU_7 (right) which later stood for the word „eating.“ In proto-cuneiform writing, however, this sign combination was exclusively used to represent a certain type of administrative activity related to the disbursement of rations.

Deciphering pictographs. Here: NINDA (bread, ration) and GU7 (to eat, distribute ration) (Source: **Damerow/1999**)



Interpreting pictographs in early accounting texts. This tablet, formerly from the Erlenmayer Collection appears to have been part of the administrative archive of a production unit concerned with the distribution of beer and the ingredients used in beer brewing (unprocessed grain emmer and barley, malt, coarse-ground barley groats). The primary administrative activity in archaic Mesopotamia was of grain storage and distribution, and these are by far have the greatest number of accounts in Uruk. (**Englund/2001**,p.3) (Source: Tablet **MSVO 3, 02** (3 columns). **Interpretation**. Publictions: **Nissen/1993** frontspiece, p.42, **Englund/1998**)

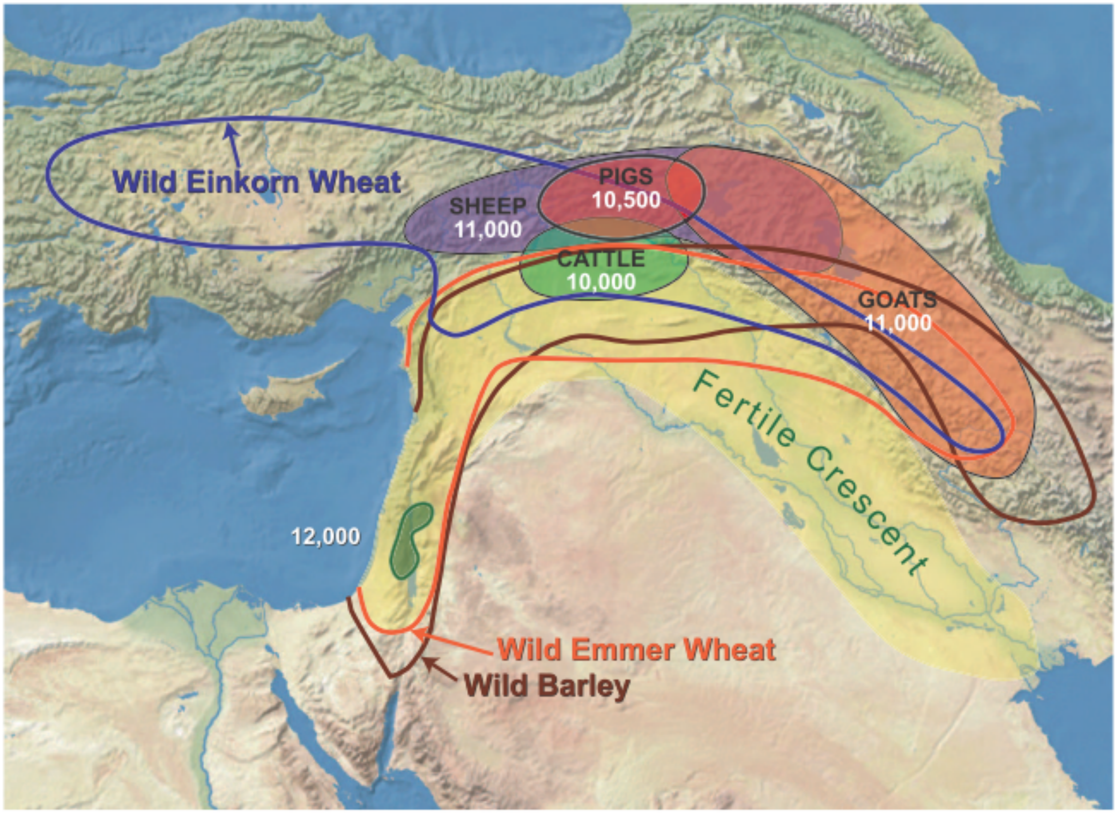
Let us now leave 3,200 BCE, the dawn of writing and move back further into the previous period, when the mathematics was developed that the scribes of Uruk and Susa would later capture.

2. Mathematical practice in the transition between Neolithic to Chalcolithic (Ubaid period): evidence from 6,000 BCE

Between 6,000 BCE and 4,000 BCE (8000 to 6,000 years ago) there is evidence of (1) painted pottery showing elaborate designs using sophisticated symmetries, and (2) layout of prestige buildings (eg temples, shrines) showing architectural competency in geometric design (parallel lines, application of right triangle ratios such as 3:4:5 and 5:12:13, use of squares and rectangles) and precise build execution with support for use of a standardized length measure. Tokens of the kind shown to have represented number, have been found dating back to c.7000-8000 BCE.

Anthropological context

The formation of settled society occurred from 12,000 to 10,000 BCE, with evidence for the deliberate cultivation of crops occurring c.9,000 BCE. This coincides with the time after the last ice-age receded from the Near East (c.12,000 BCE). These were initially small Neolithic settlements that combined cultivation of crops and management of domestic livestock (primarily sheep and goats) within an egalitarian social structure. [Charvat/2002]



The neolithic revolution that followed the retreat of the 4th glacial ice age ca 12,000 years ago.

Counting and keeping time, shape and symmetry in craft, practical matters of building and measuring, all start to appear in the archaeological record in the Near East as humanity was progressing beyond the initial stages of settled life, when sufficient agricultural surplus allowed practitioners to specialize and refine their craft and develop the technologies they used. This was a time of practitioner level mathematical knowledge, what Hoyrup describes as sub-scientific, learned “on the job”, in terms of procedures, in apprenticeship arangements. [Hoyrup/1988], [Hoyrup/1989], [Hoyrup/1994]. The results were quite remarkable and are part of the documented acceleration in Neolithic cultural sophistication. [Charvat/2002]

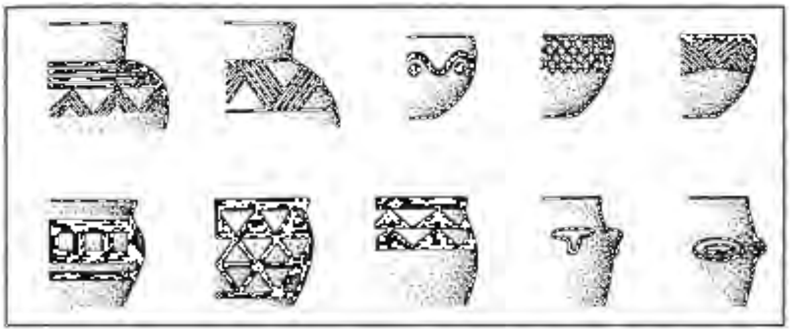
By c.6000 BCE, we see a clear shift into the transitional Neolithic-Chalcolithic Ubaid period culture, with larger settlement sizes, semi-permanent dwellings, further specialization in crafts, and emerging evidence of hierarchical social status. [**Charvat/2002**] (See **Appendix 5** for more details on what life was like then.)

Evidence for sub-scientific, practitioner level mathematical understanding can be found in the artifacts of Neolithic life: designs in pottery showing geometric regularity and the exploration of geometric patterns; building layouts showing an understanding of form, symmetry, composition; an understanding of seasonal regularity and calendarized activities: migration, planting, harvesting, all of which required reasonable mastery of the solar calendar (without which seasonal regularity is impossible); and number, which is required in cooperative behaviour: equitable distribution gains from hunt or harvest, planning for the retention of sufficient seed for sowing next season's crop, and trade/exchange across increasingly longer distances. All of these have socio-anthropological-archaeological evidence in the period between 8000-4000 BCE. We may thus pull backwards the date of the development of mathematical understanding to this period from c.9,000 to 6,000 BCE, i.e. from the period of the deliberate cultivation of crops and management of small livestock (sheep, goas) to the period of sophisticated Neolithic practitioner technology within larger permanent settlements with longer distance trade and hierarchical organization.

Let's look at each:

(1) painted pottery dating from 6,000-4,000 BCE show designs that use complex mathematical symmetries, and rotational frieze patterns, providing evidence for strong geometrical stylisation [**RobsonSelin/2000**].

Hassuna culture: painted and applique designs on pottery from the Yarimtepe I site (in Iran):



Painted pottery from Hassuna culture Yarimtepe I site dated c6500-5000 BCE. (Source: **Charvat/2002** p.23)

Samarran culture: Pottery from Samarra from 6000 BCE-4000 BCE show confidence in geometrical form:

Samarran pottery, 6000 BCE-4000 BCE (Source: **Charvat/2002**, p.35))

In the photo below of the Samarra Bowl (c.4000 BCE), we see:

> "Four stylised herons catch fish in their mouths while eight fish circle round them. An outer band of stepped lines moves outwards, countering the swirling effect of the animal figures." [**RobsonSelin/2000**]

Samarra Bowl (Pergamon Museum, Berlin), 4,000 BCE.
Painted pottery during the Ubaid period showing strong geometric stylisation (Source: **Wikipedia**, **RobsonSelin/2000**). Note Robson's image appears reversed from the Wikipedia .photo, making the flow clockwise instead of counter-clockwise

(2) analysis of **nine successive temple layouts** at **Eridu (first Sumerian city** mentioned in the King List) from Temples XVII c.5750 BCE through Temple VI, and comparison to other Ubaid period sites (6,500 BCE – 3800 BCE) show an architectural discipline in which prestige and communical buildings began to be laid out with increasing sophistication resulting in the use of modules with dimensions suggesting the use of a standardized length measure (Ubaid cubit of 0.72cm) as well as knowledge of side ratios of right triangles (3:4:5, and 5:12:13). [**Kubba/1990**], [**Forest/1991**] makes similar findings at Tell Hammam et-Turkman, **Soudipour/2007**.

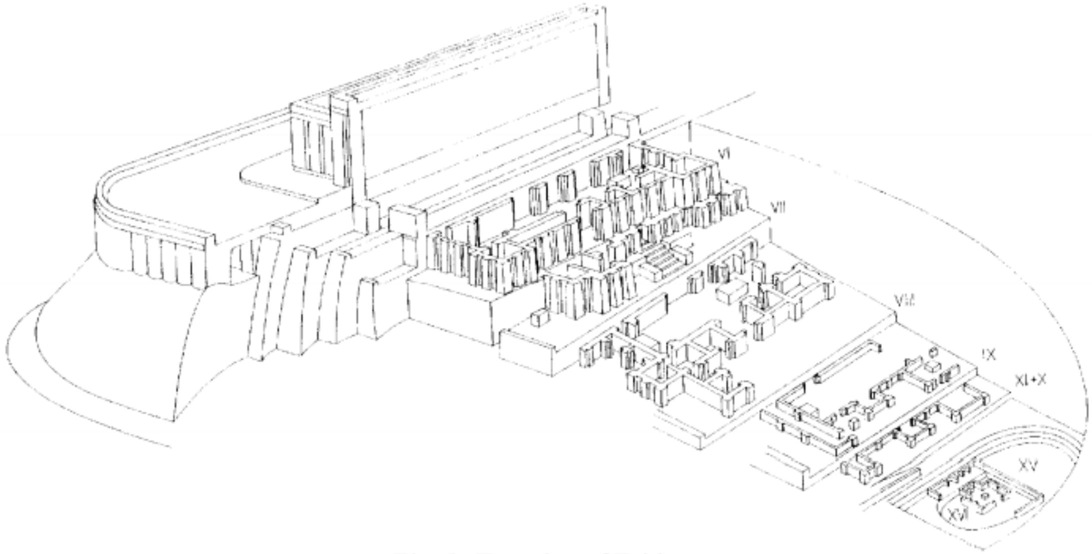
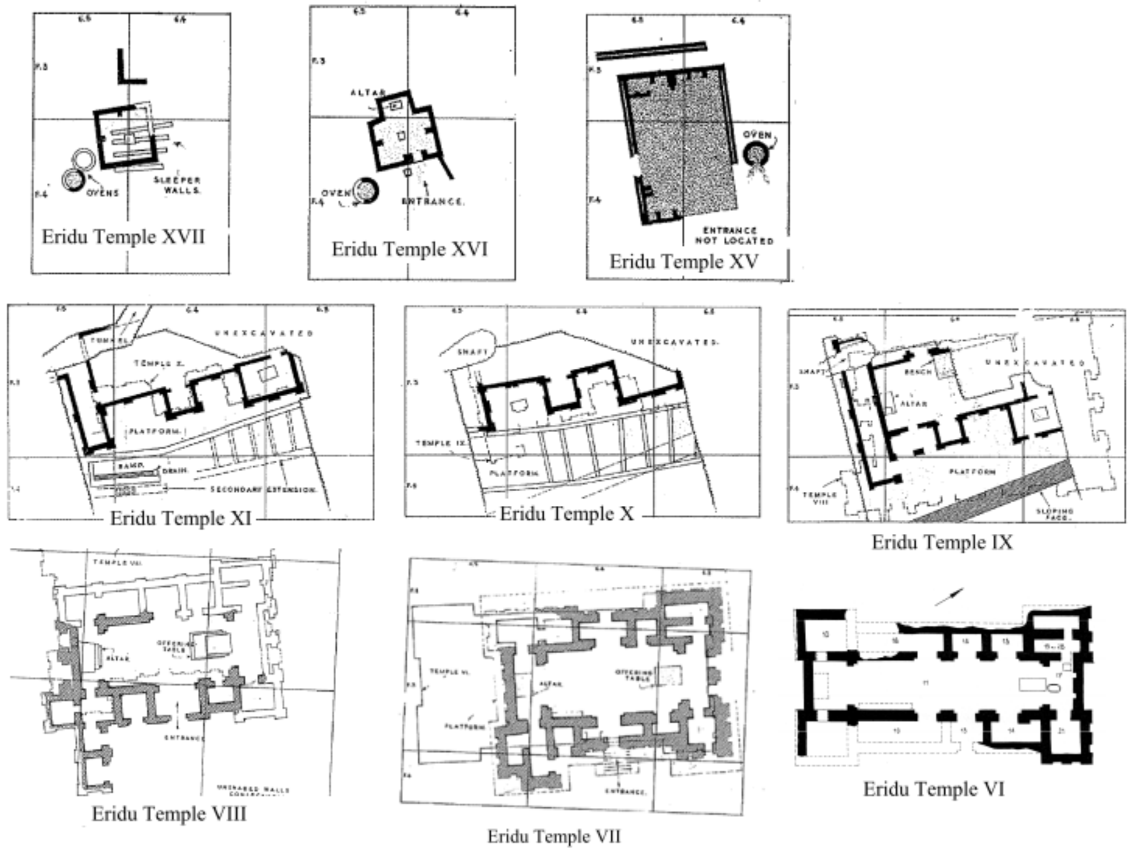


Fig. 3: Temples of Eridu

Temple layers at Eridu, built one on top of another over the course of 3000 years (5570 BCE through Ur III dynasty 2000 BCE)

Source: **Soudipour/2007**, p.100



Nine floorplans from Eridu temples c.5570 BCE onward, showing increasing architectural sophistication until Temple VI with a distinctly mathematical layout (see Kubba 1990). Source: **Soudipour/2007**, p.101-102

In addition to geometry, there appears to be some significant master builder experience involved even in the choice of orientation of the temple layout to maximze sunlight. From the earliest temple site (Temple XVII), all the buildings have a fixed orientation with corners at compass points N,E,S,W, creating a northwest-southeast axis.

> "The fact that buildings were built in such a way that their corners were made to face the central axis indicates an excellent knowledge of climatic matters. When the corners of the building face the north-south axis, the four sides of the building receive maximum sunlight, the northeastern and south eastern wall receive the sunlight until midday and the northwestern and the southwestern walls receive the sunlight after midday. Thus, all four sides of the building receive sunlight daily" (Youkana, 1997: 63)"

Why have complex temple building activities not yielded written mathematical evidence? Essentially, it is due to what could be considered guilds in the early society. The guild of master builders had their own domain knowledge. The guild of temple administrators (SANGA) and chief administrators of a city (EN) had their own domain knowledge, essentially that of a quartermaster (senior individual supervising stores and distributing supplies and equipment) crossed with bookkeeper/accountant (recording of financial transations, including purchases, sales, receipts, payments). The invention of written mathematics was in the guild of the quatermaster/bookkeeper in the context of running an increasingly complex temple economy. The builders had no such practical pressure/problem for which written mathematics was needed.

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Jens Hoyrup explains: "Temple building must have involved a fair measure of practical geometrical knowledge, but evidence from later times suggests that this knowledge was the posession of master builders and did not communicate with the mathematics of the literate managers." (**Hoyrup/2011** p.4)

Sophisticated naive geometrical knowledge and its associated geometrical algebra appears more or less fully clothed in scribal mathematics at the end of the Ur III period in the northern periphery of the Ur III empire (Eshnunna, c.2030 BCE) leading eventually to the sophisticated Old Babylonian mathematics that Neugebauer was able to decipher. [Hoyrup/1985], [Hoyrup/1990], [Hoyrup/1993], [Hoyrup/1996], [Hoyrup/2002]. The traditional practitioner, guild-level knowledge of geometry understood by the master builders and field surveyors (rope stretchers) appears to have remained unchanged through to Islamic mathematics when al-Khwarizmi documented it as the science of the al-jebr guild, in his masterwork "Algebra", through which this knowledge made its way to medieval Europe, persisting still in its naive geometrical form until the 1500s (Pacioli), finally dying in the work of Viete who transformed algebra into a symbolical instead of rhetorical discipline in the 1590s. [Hoyrup/1994] When was it discovered? Again, we can give no firm date, but we can give a range, from 6,000 BCE during the extensive temple building and agricultural organization of larger settlements during the Ubaid period, through to 2,030 BCE when it first appears in the written record.

The interaction between culture and mathematical development

By the end of the Ubaid and at the start of the Uruk period, settlements were for the first time able to generate significant food surpluses through centralized management of labor resources directed to building large-scale irrigation systems to improve food output. The resulting economic and social changes, the transition from settlements to city-states, the rise of an established urban elite, and the shift to a redestributive, centrally controlled temple-economy drove the use of tokens as accounting devices, as we have seen above. [**Charvat/2002**], [**Niemi/2016**]

In this and the previous period, what we have seen is that mathematical practice has arisen within a social context. It has been developed (invented?) and perfected within that social context for an application with a social purpose (accounting, recording of commercial transactions, state management of labor and food surpluses, design of prestige buildings, or the manufacture of status goods). Through its use, mathematics then affects and in many cases transforms the culture it arises within, and shifts it in new directions, which in turn affect the direction of further mathematical practice. See [**Hoyrup/1991**] and [**Robson/2008**] for examples of how the culture of the scribal schools varied from Uruk to Akkad to Ur to **Hammurabi**'s Babylon to the fall of Babylon, a period covering 1500 years.

3. Into the Paleolith: limitations of direct archaeological evidence, and a look at controversial claims dating mathematical practice to 70,000 BCE.

Before 10,000 BCE, there are a few isolated finds with controversial mathematical or calendrical interpretations, but nothing convincing. For example, we exclude the Ishango, Lebombo, and Wolf bones, and exclude also the engraved ochre from Blombos Cave. The argument for their mathematical *nature (Marschak) is based on close reading of their markings and association with tallies, prime number groupings, or calendric tabulation. But the notches on the bones (for example) could have non-mathematical hypotheses, e.g. to improve their grip for use as a tool or weapon. These finds do not pass Newton's test (1713) against speculation: “**Hypothesis non fingo”** meaning “[I am certain if] I need feign no hypotheses!” [Walsh/2010].*

Can we find direct evidence of mathematical practice in the Paleolithic before humanity became settled?

There are two problems with older archaeological evidence: the first problem is that many materials that may have been part of mathematical practice are bio-degradable and would not have survived (e.g. markings on sand with a stick, tallies on wood). Those that could survive (pebbles, bones) lack any cultural context to confirm mathematical usage. For example: notches on bones could suggest tallying, arithmetic, an understanding of prime numbers, or pre-historic calendar cf. [Marshack/1971]. But they could equally well be explained by non-mathematical intent, e.g. to improve the grip of the object used as a hammer or club [Elkins/1996].

The second problem is that the paeleolithic finds are isolated geographically (Ishango bone in Uganda, Lebombo bone in Swaziland, and Wolf bone in the Czech Republic) and in time (dated between 18,000 and 35,000 BCE). There is little to no archaeological context of the finds that would suggest mathematical intention, which therefore relies entirely on interpretation of marks which remain tenuous and highly controversial (cf. Claim [Marschack/1971] and rebuttal [Elkins/1996]; Claim [Huylebrouke/1996], and rebuttal [Keller/2010]).

Similar problems beset the interpretation of an engraved red ochre lump from S.Africa dated to c.70,000 BCE. Suggestions of geometric decoration are hard to conclude without repetition or other context. They could also have been attempts at cleaning the point of a blade, or use as a cutting board, or scrapings to release coloured powder from the ochre for pigment dye.

Notched bone (Ishango) and engraved red ochre (Blombos cave). There is no context that indicates whether the markings have meaning.

These are the circumstances surrounding all paleolithic artifacts discovered so far to which a mathematical culture has been ascribed. We simply do not have enough archaeological context on why or what they were carved for in order to interpret them. Unfortunately, speculative interpretations have made their way into news media and non-specialist literature covering ethno-mathematics and, regrettably, even into textbooks on mathematics history. The interpretations have ranged from lunar calendars and fertility tallies, to multiplication tables and prime number lists. As an example, textbook historian David Burton follows Marshack and represents a current enthusiastic popularization when he writes of the Ishango bone: "It had been used for reckoning time "in sequences of numbers that agree with the number of days included in successive phases of the moon." [Burton/1982] [Burton/1982], [Huyle/1996], [PletserHuyle/1999].

More critical recent scholarship has drawn important cautions: [Elkins/1996] takes apart Marshack's microscopic readings of notched bone and highlights the repeated unjustified leaps in going from evidence to conclusion. [Keller/2010] summarizes:

> "The siren song of mathematical illusion is never far away when it comes to prehistoric artifacts. A notch may be nothing more than a mark [unless] one is obsessed with arithmetic [in which case the sign joins] the common denominator of all the ethnographic artifacts of this kind [showing] item by item [bijective] symmetry between objects and signs. Faced with the raw artifacts of prehistory, it is impossible to know … whether the markings are decorative or not, and if they are not, whether we are dealing with an artificial memory system." (Keller/2010)

4. Indirect evidence: the rise of symbolical capability in humans from 315,000 years ago

Turning to indirect evidence, we ask: ***when did the capability emerge for mathematical thinking (number, shape, time, change)?*** *As early as 230kya,* ***the archaeological record (Omo 1 site in the Ethiopian side of the Rift Valley)*** *shows changes in human species as anatomically modern humans (H. sapiens) diverged from Homo Erectus. New finds in* ***Morocco (Jebel Irhoud site), push this date back to 315kya*** *(though some contest whether the latest finds are H. Sapiens). Evidence of complex behaviour (ritual burial of the dead, cooperative hunting, the controlled use of fire, language capability) suggests the capacity for symbolical thinking that would be a prerequisite for any sort of mathematical practice (counting, bijection, keeping a tally, measuring, symmetry, or abstract artistic design).*

In light of the previous section, we do not currently have direct conclusive evidence of mathematical practice from before 10,000 BCE when the last ice age receded. But if we modify the question to inquire when humanity developed the symbolic capability to support numeracy, then we can go back further to 315,000 years ago (note this is the last 9% of human presence on earth, which stretches back 3.5 million years).

Archaeological evidence shows that intelligence, communication, and social living stretch back to 315,000 years ago (Middle Pleistocene), when humans had already evolved into what is essentially their modern form, Homo Sapiens, and were using speech, tools, fire for warmth and cooking, were hunting large adult animals, and had diversified into all of the major races. By the time of the fourth glacial advance 100,000 years ago (Upper Pleistocene), anatomically modern humans (H. Sapiens) dressed and sewed skins, were able to live beyond the frost line, had a culture of arts and crafts and a ceremonial society that buried the dead and showed solicitude to the aged and maimed. (See **Appendix 4** for details of life in the paleolithic to the start of the neolithic.) Presumably, then, there would already have been utility in comparing, for example, the number of men in a hostile encampment with those in the home group, and in communicating this numerical information for group action. Similarly, a builder or toolmaker needing material for a particular purpose would have needed to specify dimensions, even if roughly. An elder needing to know how long a hunting party had been absent before setting off to investigate would have needed to mark time.

Until early in the current century, the prevailing opinion was that humankind developed symbolic capability between 50,000 year ago and 315,000, coincident with the emergence of anatomically modern humans (H. Sapiens). This was based on:
(1) the discovery of the earliest human art (cave paintings, jewelry/decorative power),
(2) anthropological evidence of ritual burial of the dead,
(3) anthropological evidence of cooperative hunting which presumes the ability to communicate intentionally and with precision,
(4) the practice of language, indicated by earliest presence of the human version of the FOXP2 gene which regulates learning and complex speech, combined with the assumption that (a) the ability to speak implies that speech and language occurred, and (b) that any language, no matter how primitive, must be symbolic and include at least a rudimentary number concept (e.g. one-two-many, or even one-many). The correctness of this last assumption was justified by the evidence of all known primitive languages encountered before the 1970s) (cf. [Conant/1897], [Smith/Ginsburg/1937] and [Gullberg/1997])

These views have changed in the past 20 years following extensive analysis and study of the Piraha people of the Brazilian Amazon discovered by Western sociologists and anthropologists in the 1970s, whose language surprisingly has no numerical concepts at all [Piraha/2006]. The Piraha (both the people and their language) provide observational evidence that there can exist a state of being in which symbolic capability is present but numerical capability in language and culture does not result. [C.Everett/2016] [DL.Everett/2018]

Language and the Number Concept

Speech had previously been viewed as a proxy indicator of numeracy, since before the Piraha every language and culture previously known had numeric concepts. [Conant/1896]: “We know of no language in which the suggestion of number does not appear, and we must admit that the words which give expression to the number sense would be among the early words to be formed in any language.” The unusual language and culture of the Piraha people has no numbers, not even the “one”-“two”-“many” pattern found in other primitive languages. They become the first known counter-example, in the process changing our view of what language is and how it may have evolved. [SG/1937], [Piraha/2006], [Piraha/2007], [Gordon/2004], [FEFG/2008], [EM/2012]

Studies of the Piraha suggest that numerical capability appears to require three things: (1) the capability for symbolic thought (e.g. grasping the notion of bijection, which underpins discrete comparison); (2) a mechanism to keep the count (e.g. fingers, marks/notches, pebbles, or linguistic counting words), and, taken for granted before the Piraha, and, most importantly perhaps, (3) a culture that assigns *value* to planning, forethought, and material acquisition, all of which are supported by numeracy. The Piraha culture rejects planning, forethought, and is non-materialistic to the extreme, resulting in placing no value for number in their culture. As a result, not only have they not developed any mechanisms for counting, but they actively resist the learning and retention of these mechanisms when they are introduced to them, despite being able learners of other things [Gordon/2004], [Frank.DLEverett/2008], [C.Everett/2016].

This places the development of mathematical practice within cultural context once the fundamental neurological ability for symbolical thought exists. While one may indeed grasp the notion of bijection, without a mechanism to keep a precise tally one cannot actually count, only match. How the tally itself is made is less important and can take many forms: visually by using fingers of the hands or creating marks or notches, physically by collecting pebbles or other tokens or calculi, or linguistically using by words and/or signs. But without valuing the act/outcome of counting/accounting/planning, the Piraha example shows that humans essentially fall back on what appears to be a biologically innate analogue number sense that is also present in animals, birds, and even some reptiles, but which decreases in precision as magnitudes get larger. This is why animal counting degrades quickly beyond four or five. [Gordon/2004] [Everett/2012], [Dehaene/1997]

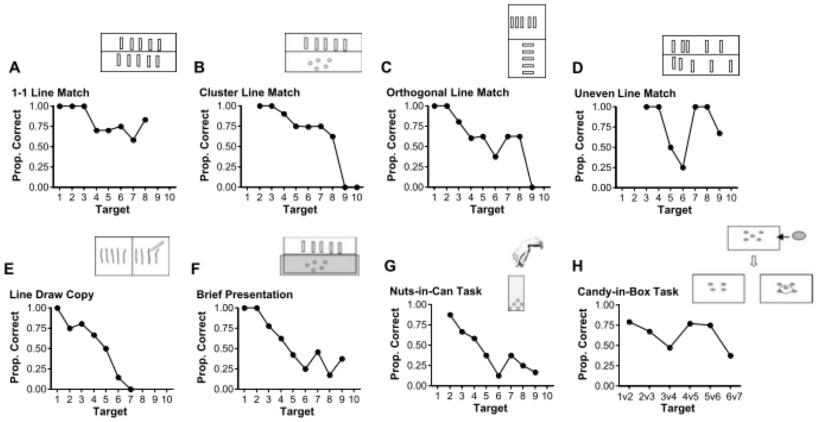


Fig. 1. Results of number tasks with Pirahã villagers ($n = 7$). Rectangles indicate AA batteries (5.0 cm by 1.4 cm), and circles indicate ground nuts. Center line indicates a stick between the author's example array (below the line) and the participant's attempt to "make it the same" (above the line). Tasks A through D required the participant to match the lower array presented by the author using a line of batteries; task E was similar, but involved the unfamiliar task of copying lines drawn on paper; task F was a matching task where the participant saw the numerical display for only about 1 s before it was hidden behind a screen; task G involved putting nuts into a can and withdrawing them one by one; (participants responded after each withdrawal as to whether the can still contained nuts or was empty); task H involved placing candy inside a box with a number of fish drawn on the lid (this was then hidden and brought out again with another box with one more or one less fish on the lid, and participants had to choose which box contained the candy).

DER SPIEGEL

Graphic: The Piraha people's home

Experiments on Piraha numeracy (Source: Gordon/2004); (right) Piraha homeland (Source: Der Spiegal/2006)

The evidence for speech

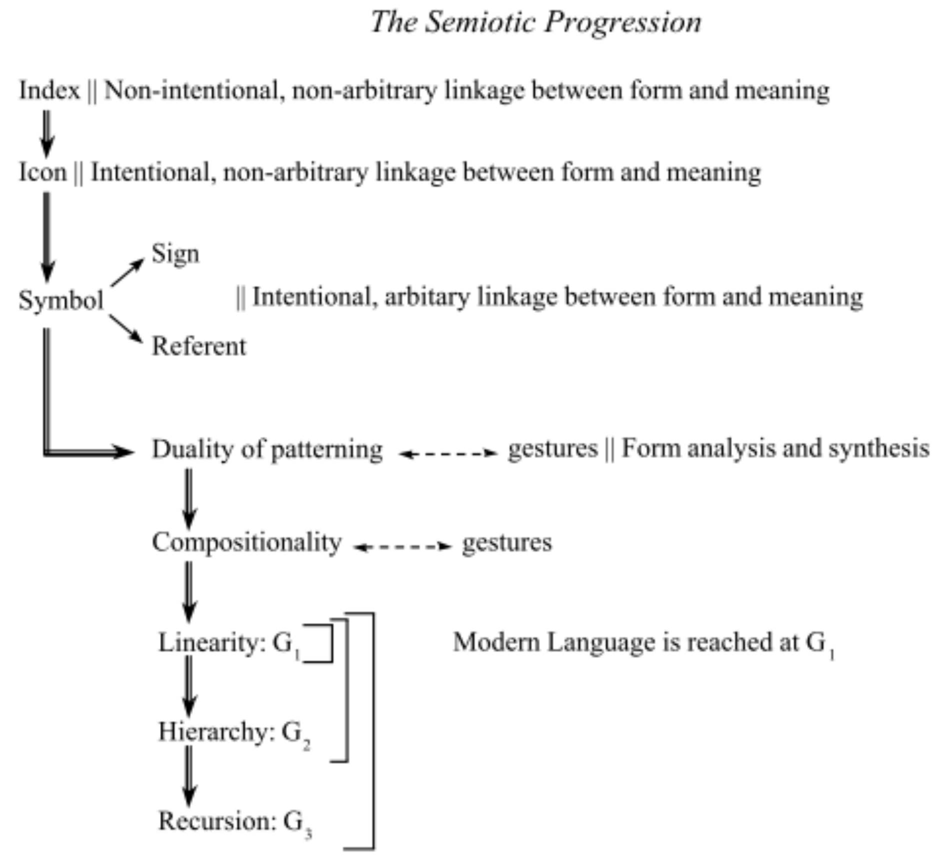
What evidence exists for speech? Genomic investigations into speech defects have identified the **FOXP2 gene** as a critical link to and enabling factor of speech control. Absence leads to non-viability, reduction leads to significant vocal disability. While the FOXP2 gene is expressed in birds, mice, primates, and humans, the human variation is different from all the others. The modern functioning version has been present in humans between 120,000-260,000 years ago, either the last common ancestor of Neanderthals and Homo sapiens, or specific to Homo sapiens. And so we form the basis of the argument: the capability for complex vocalization means the ability to realize speech and language. From language comes symbolism. Within a culture that valued planning, control, and materialism, the number concept can develop. All of the pieces for were therefore in place by 230,000 years ago (FOXP2 gene by 260,000 years ago, evidence of social/cooperative living by 315,000 years ago).

5. Paleo-anthropological evidence from 2.3 million years ago and the semiotic model of human conceptual development

Fossil records from Lokalalei, Kenya (near Lake Turkana) have shown the emergence of sophisticated stone knapping techniques 2.3 million years ago among early hominids (unclear whether Australopithecus or Homo Erectus) [Delagnes,Roche/2005]. If these were the simple split-stone variety (one strike, one split, use the edges that result), it would not be a surprise since the simpler Oldowan stone tool culture dates already from 2.6mya. What is surprising is that these stone tools from Lokalalei were made using the complex multi-strike techniques for forming blades from a carefully selected blank flint core using a sequence of strikes to create a razor sharp tapered edge. This technique requires considerable experience with how stone shatters as well as advance planning requiring to visualize how the sequence will work to create the tapered edges. Except for the Lokalalei site, such tools are only found in the fossil record from 1.7mya onward (Acheulean culture), some 600,000 years later. Looking at C.S Peirce's semiotic model for conceptual and linguistic development (see below), we have in the Lokalalei stone knapping process two indications of early hominids having reached Stage 3 symbolic behaviour: the considerable planning requirements to shape the blade, and the cultural transmission (teaching) of the technique. This provides a ***terminus ante quem*** *(latest date) of 2.3 million years ago for abstract symbolic thought.*

Using C.S. Peirce's semiotic progression (index, icon, symbol) for evaluating linguistic and conceptual development.

C.S. Peirce's semiotic model posits that conceptual and linguistic development pass through 3 stages: physical/index, associated icon, and abstract symbolic (cf. [Everett/2017]). This makes it a useful model for empirically situating any given activity and placing it within the 3 sequential states.



Semiotic Progression model of language acquisition, following C.S. Peirce (Source: Everett DL, 2016, p.18)

Index conditioning (Stage 1) is the ability of creatures with a nervous system to perceive an “index” (physical stimulus) and produce an appropriate response (e.g. recoil from a hiss, be wary of yellow and black insects, recognize footprints or smells). This capability gives intelligent animals and humans have the ability to recognize and respond to sounds (bell, word, clap, sound of water suggesting presence of water) or visual cues (hand signal, position of ears, baring of teeth, etc.) or any of the other senses. Both the perception of stimulus and the pathways for response are biological and neurological. This is what allows a variety of animals, birds, and reptiles to possess a number sense and to perceive shape, time, and change (the cognitive precursors of mathematics). Memory, adaptation, and trained learning are forms of index conditioning.

Icon communication (Stage 2) involves the intentional use of signs (“icons”) chosen because of their close association with the intended physical meaning (e.g. smoke for fire, a figurine for motherhood, a stick drawing for a person, or a footprint or smell for the creature that caused it). Stage 2 is the understanding and use of “icons” which are associations intentionally chosen to represent physical phenomena (e.g. picture of cow, picture of fire, emojis, etc.). The majority of animals have not been found to be able to reach stage 2, with the exception of some primates, but even when they do show the ability to understand icons, **they do not show the ability to take these learnings back to their communication with each other**.

Symbolic communication (Stage 3) involves the intentional selection of arbitrary signs whose meaning is established by cultural convention (e.g. male/female signs, traffic light colors, arbitrary gestures, tallies, arithmetic signs, numerals, etc.). Stage 3 is the use of abstract symbols, i.e. signs that are purely arbitrary and require establishment by cultural convention in order to interpret. Examples include symbols such as $ or £ or traffic light green for go/red for stop, a heart sign for love, sign language, alphabet, logograms, words/names, and NUMBERS. [1]

Over the past 20 years, the fieldwork approach to linguistics has challenged the Chomskian theory of language acquisition in early humans. The importance of the Piraha to theories about prehistoric language development and numeracy is that they provide field evidence that anatomically modern humans have not reached Stage 3 in the semiotic progress, remaining at Stage 2, apparently by cultural choice.

Inferring human capability from the fossil record

The making of effective bladed stone tools for cutting and scraping requires the ability to think abstractly and to conceptualize and foresee the consequence of a certain way of striking the stone to create a certain kind of fracture. When done expertly, the resulting blades are sharper than razors, sharper than surgical knives. Indeed, in the modern era of medicine, before super thin metal blades could be produced, surgical knives were indeed made from expertly knapped flint.

Stone tool-making among early humans is considered to be an aspect of transmitted culture based on the ability to consistently produce such artefacts through broad geographic regions and through time. The first of these cultures, the Oldowan c.2.6 mya produced simple split stones, but already this was enough to show that communal sharing of knowledge had developed in early hominins to enable this method of production to continue unbroken for the next 900,000 years (till 1.7mya). At this point, it was replaced by the improved Acheulean tool-making method of the late Homo Erectus period (1.7 mya). Evidence from [Delagnes,Roche/2005] have shown that similar exceptional stone knapping capability to produce bladed tools were present at the Lokalalei site 2.3 million years ago. This kind of complex stone knapping was **highly dangerous as evidence from modern lithic workers attempting to reconstruct the old ways have found out** (sharp fragments flying off at high velocity, with the makers wearing no gloves, no shoes, no protective eyewear, no trousers).

The archaeological data and evidence of sophisticated bladed stone tool creation at Lokalalei provides two arguments for humankind reaching Stage 3 symbolic capability at least by 2.3 million years ago, almost 14x earlier than the earliest Chomskian estimates: the capability itself, and the ability to transmit that knowledge.

Oldowan Tools

Acheulean tools:

Stone Tool Cultures of early Hominids: Oldowan (2.6mya) and Acheulean (1.6mya)
(Source: EverettDL/2016)

6. Culture transmission and the cognitive precursors for mathematics in animals: back to 260 million years ago

How early does culture transmission in animals manifest itself? When does the analog perception of number appear in animals? Around 260 million years ago, neurological and biological evolution had progressed to the last common ancestor of birds and mammals, a reptile that shared the brain circuitry of both and which underpins the index/response mechanism that gives an analog number sense (small numbers) to animals, birds, and monitor lizards (reptiles).

Culture transmission and iconic or symbolic association are necessary conditions for mathematical understanding. Cultural transmission in order to teach the understanding and use of complex ideas, tools, or technology. Iconic/symbolic capability to be able to recognize and work with quantity, form, and the perception of change.

The presence of culture (socially transferred knowledge) has been observed in chimpanzees (**Oct 2009 study**) who share tools and teach tool use (**Oct 2016 study**).

Adult members of the Piraha tribe use what appears to be a biologically innate analogue number sense that is also present in animals, birds, and even some reptiles. The precision of this number sense decreases as magnitudes get larger, and explains why animal counting accuracy degrades quickly beyond four or five, cf. [Everett/2012], [Dehaene/1997].

Do number sense (but perhaps not measurement or counting per se) and the perception of shape and change (but perhaps not their description or communication) occur outside human species? Investigations have found evidence of number sense in animals (birds, dogs, monkeys, dolphins). Perception of the passage of time, the ability to distinguish one from many (in particular, quantities other than two), and the ability to distinguish shapes from each other, have all been documented in various animals. [Koehler/1950]

> "A man was anxious to shoot a crow. To deceive this suspicious bird, the plan was hit upon of sending two men to the watchhouse, one of whom passed on, while the other remained; but the crow counted and kept her distance. The next day three went, and again she perceived that only two retired. In fine, it was found necessary to send five or six men to the watch house to put her out in her calculation. The crow, thinking that this number of men had passed by, lost no time in returning.' From this he inferred that crows could count up to four." John Lubbock, _Nature_, Vol. XXXIII. p. 45., from [**Conant/1896**].

> "A nightingale which was said to count up to three. Every day he gave it three mealworms, one at a time. When it had finished one it returned for another, but after the third it knew that the feast was over…." Lichtenberg, _Nature_, Vol. XXXIII. p. 45., from [**Conant/1896**].

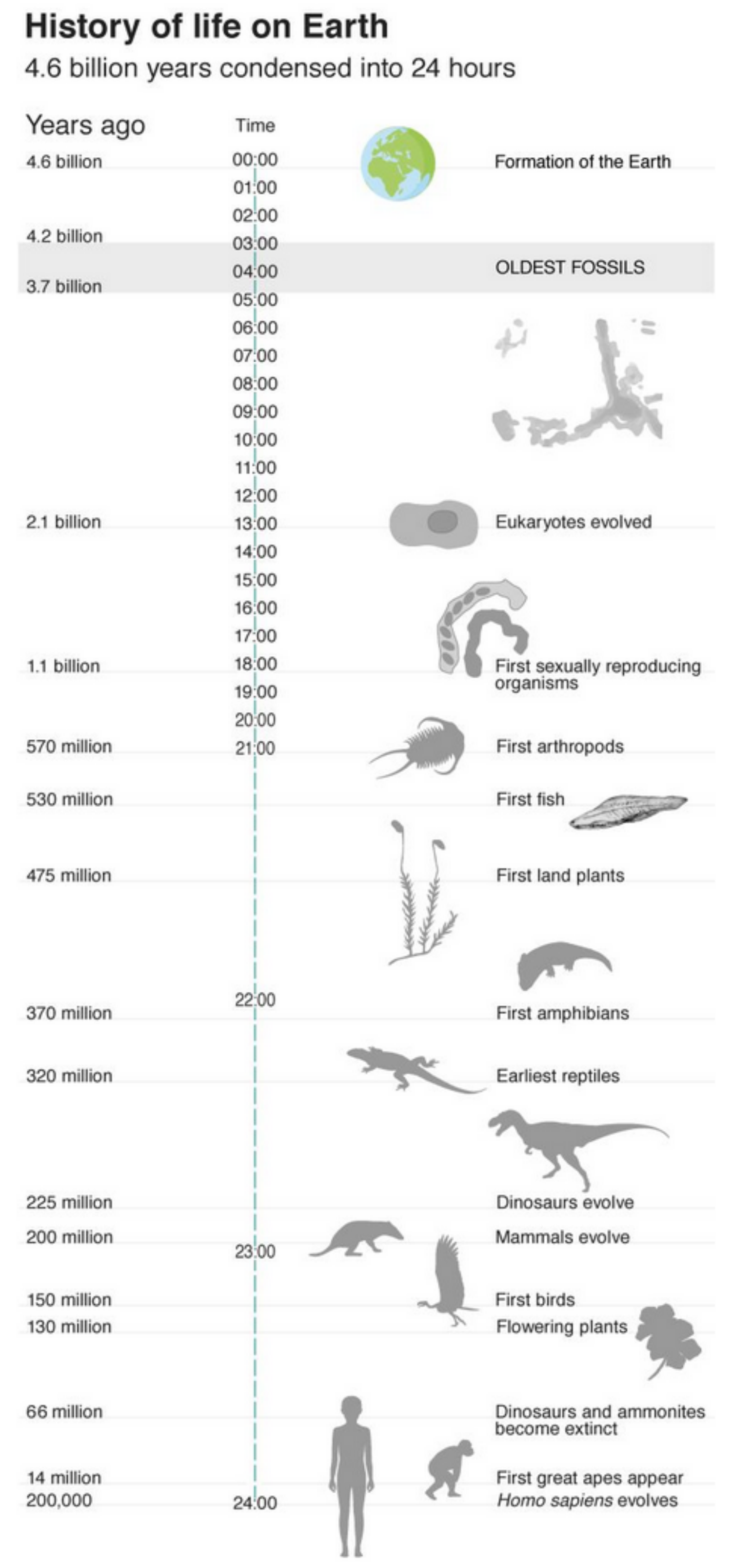
> "Dinah, my spaniel, … was overlooking half a dozen of her new-born puppies, which had been removed two or three times from her, and her anxiety was excessive, as she tried to find out if they were all present, or if any were still missing. She kept puzzling and running her eyes over them backwards and forwards, but could not satisfy herself. She evidently had a vague notion of counting, but the figure was too large for her brain." Galton, _Nature_, Vol. XXXIII. p. 45, from [**Conant/1896**].

The semiotic perspective highlights that perception and response to indexes (unintentional physical associations) is common to all living things that can think (sense their environment and choose a response).

If we ask when this analogue number sense may have developed in animals, this takes us back much further. Primates go back to 13 million years ago, birds to 150 million years ago, mammals to 220 million years ago, taking us back to the last common ancestor of birds and mammals having the same brain structure, which would have been a **stem reptile c.260m years ago**.

Dating the capability for mathematical cognition then becomes a question of the timeline of intelligent, perceptive life itself. The intelligent tree-dwelling primates of 13 million years ago likely had the mental capacity for cognition of the precursors of mathematics. Are animals able to progress from the lowest rung (indexes) of Peirce's 3-step evolution to the next rung (icons)? Experiments have shown that animals can proceed from icon to correct decision (this is learning and cataloguing new indexes e.g. Pavlov's dog, trained monkey, crow), the challenge not yet demonstrated (as far as I am aware) is of an animal taking the stimulus, and picking the right descriptive icon, i.e. classification. Similarly, I am not aware of a non-human animal intentionally adopting a completely arbitrary symbol or sign, whose interpreted meanings need to be established as part of a cultural convention, unless we take animal language to be such an example.

Can we pin an upper time limit to the existence of a brain capable of perceiving number, shape, change, time and responding? Studies have shown that an analog number sense (recognition of numbers smaller than 6 in an analog/imperfect way) exists in mammals (dogs, monkeys), birds (parrots, crows) and even reptiles (monitor lizards, [Pianka/King 2004, Murphy 2019]). Neurological studies have established that in mammals, it is the mammalian neocortex (evolved 220 mya) that is the seat of complex cognitive functions such as sensory perception, spatial reasoning, learning and memory, decision making, motor control, and conceptual thinking. In birds, it is the DVR (dorsal vernicular ridge) that provides neocortical-like functioning. Both the neocortex and the DVR have been found to develop out of the same region in the embryonic brain. This points back to the **neurological circuitry of a common ancestor of mammals, birds, and monitor lizards, i.e. a stem reptile (amniote)** existing some 260 million years ago.



Timeline of Life on Earth. First birds at 150mya. Reptiles as the last common ancestor of birds and animals c.260mya (Source: Wikipedia)

7. Conclusions

How far back do we have evidence for mathematical practice? What about the cognitive, social, and cultural aspects needed for its cognitive precursors?

We have seen in this article that:

Direct evidence for mathematical knowledge exists from c.6,000 BCE or 8,000 years ago.

Humans developed the capability for abstract thought around 2.3 million ago which continued to drive innovation through to 315,000 years ago, by which point the last of the major evolutionary changes leading to anatomically modern humans was complete.

The intrinsic ability to perceive number, size, shape, time, and change trace back beyond humans themselves and into mammals and birds, back some 260 million years ago, to the last common ancestor of mammals and birds.

If we tell the story in the correct order, it looks like this:

1. c.220m years ago the mammalian neocortex, and by 150m years ago the dorsal vernicular ridge (DVR) in birds had evolved, and these are the neurological seats of cognitive recognition the index/response mechanism that underlies the analog number sense (for small numbers) that is documented in mammals and birds. The number sense documented in monitor lizards (reptiles) would push the date further back to a reptilean common ancestor of mammals and birds, **a stem reptile, c.260m years ago, sharing the neurological circuitry common to both**.
2. The extinction event for land dinosaurs which occurred at 66 million years ago (mya) touched off a rapid cooling off period in global temperatures, present in geologic evidence. The disappearance of the dinosaurs led to the **proliferation of mammals** into the ecological niches vacated by the dinosaurs. Mammals it turns out, **had existed since c. 200 mya**, but had remained small, mostly nocturnal, and either tree-dwelling or burrowing, to avoid competition with the dominant dinosaurs. From 34 mya to 23 mya the Earth transitioned from a tropical world to modern ecosystems. From 23 mya to 2.6 mya the cooling continued. Primates were already living in trees by 13 million years ago, and hominids had branched off between 7.5 to 5.6 mya. At 2.6 mya, the four ice ages began (Pleistocene period) with the last glacial retreat occurring around 12,000 years ago (12 kya) and the inter-glacial warming period (holocene) beginning 10 kya. [Coon/1996]. See **Timeline (PDF) of Early Human Life, from 55mya to 5kya (Tom Conklin, 2009)**
3. Around 2.3m years ago, we see in the fossil record the first appearance of technologically complex stone knapping tool-making by early human species, that provide evidence of the progression of human-kind from index/response (shared with animals) to iconic/symbolic thinking.
4. As early as 315,000 years ago, we see in the archaeological record (**Omo 1**), changes in human species as anatomically modern humans emerge, and we have evidence of complex symbolic behaviour, which in principle could support numeracy.
5. Around 6,000 BCE (8000 years ago) there is evidence of elaborate pottery with mathematical designs, disciplined building layouts showing the use of a standardized length measure and an understanding of principles of geometry including the application of right triangles.
6. By 3,200 BCE (5200 years ago) there is indubitable evidence for mathematical practice in clay tokens and bullae ("envelopes") in Mesopotamian city states within a centralized temple economy and a scribal-statal context. This is the earliest known system of metrology (counting and measuring), of writing, and of book-keeping (accounting)

But of course, we continue to uncover more about our prehistoric past, so the story of the prehistoric origins of mathematics is undoubtedly not yet complete.

Go to Bibliography

Appendices

Appendix 1. The Essential Nature of Arithmetic

The following are extracted verbatim from the outstanding essay of A.D. Aleksandrov, A General View of Mathematics, specifically section 5, pp.17-19, on the Essential Nature of Arithmetic in [**Aleksandrov/Arithmetic1956**].

1. How did the abstract concepts of arithmetic arise and what do they reflect in the actual world?
"The concepts [of arithmetic] arose by way of abstraction as a result of the analysis and generalization of an immense amount of practical experience. They arose gradually; first came numbers connected with concrete objects, then abstract numbers, and finally the concept of number, in general, of any possible number. Each of these concepts was made possible by a combination of practical experience and preceding abstract concepts. This, by the way, is one of the fundamental laws of formation of mathematical concepts: They are brought into being by a series of successive abstractions and generalizations, each resting on a combination of experience with preceding abstract concepts. The history of the concepts of arithmetic shows how mistaken is the idealistic view that they arose from "pure thought," from "innate intuition", from "contemplation of a priori forms," or the like." [Aleksandrov/Arithmetic1956]

2. Why are the conclusions of arithmetic so convincing and unalterable?
"We see that the conclusions of arithmetic have been worked out slowly and gradually; they reflect experience accumulated in the course of unimaginably many generations and have in this way fixed themselves firmly in the mind of humankind. They have also fixed themselves in language; in the names for the numbers, in their symbols, in the constant repetition of the same operations with numbers, in their constant application to daily life. It is in this way that they have gained clarity and certainty. … What is essential here is not only the fact that they can be repeated at will but their soundness and perspicuity, which they possess in common with the relations among things in the actual world. This is the reason why the results of arithmetic are so convincing: its conclusions flow logically from its basic concepts, and both of them, the methods of logic and the concepts of arithmetic, were worked out and firmly fixed in our consciousness by [five] thousand years of practical experience on the basis of objective uniformities in the world around us." [Aleksandrov/Arithmetic1956]

3. Why has the abstract concept of number and arithmetic taken so long to arise?
"Every abstract concept, in particular the concept of number, is limited in its significance as a result of its very abstractness. In the first place, when applied to any concrete object it reflects only one aspect of the object and therefore gives only an incomplete picture of it. … It is impossible to apply arithmetic to concrete problems without first convincing ourselves that their application makes sense in the particular case. If we speak of addition, for example, and merely unite the objects in thought, then naturally no progress has been made with the objects themselves. But if we apply addition to the actual uniting of the objects, if we in fact put the objects together, for example by throwing them into a pile, in this case there

there takes place not merely abstract addition but also an actual process, and in general it may be impossible to carry it out. For example, the object may break, wild animals if placed together may tear one another apart, materials put together may enter into a chemical reaction and so the sum e.g. of a liter of water and a liter of alcohol will not yield two liters of mixture but 1.9 as a result of partial solution of the liquids, and so on. To put it briefly, truth is concrete, and it is particular important to remember this fact with respect to mathematics exactly because of its abstractness." [Aleksandrov/Arithmetic1956]

4. What forces led to the development of mathematics?
"For arithmetic, the answer is clear from history. The forces that led to the development of arithmetic were the practical needs of social life. People learned to count and to work out the concept of number. Practical life, by posing more difficult problems, necessitated symbols for numbers. These practical needs and the abstract thought arising from them exercise on each other a constant interaction. The abstract concepts provide in themselves a valuable tool for practical life and are constantly improved by their very application. Abstraction from all nonessentials uncovers the kernel of the matter and guarantees success in those cases where a decisive role is played by the properties and relations picked out and preserved by the abstraction. In the case of arithmetic, this is the quantitative relations. … This is just a particular case of a phenomenon known to everyone, namely the interaction of experience and abstract thought, of practice and theory." [Aleksandrov/Arithmetic1956]

Appendix 2: The Invention of Writing: An Advancement of Bookkeeping

1. When was writing invented?
Proto-writing first appeared at the end of the 4th millennium BCE (c.3200) in southern Mesopotamia (Uruk) and Khuzistan (Susa). This was first purely numerical recording of quantities that had previously been recorded using tokens with the commodity understood from context; then the recording of the commodity separate from the quantity using pictographs; and then by end of 4th millennium, c.3200 proto-cuneiform in Uruk where both quantity and commodity were recorded in cuneiform encoded pictographs, followed by proto-Elamite in Susa c.3000. This led to then the standardization of cuneiform pictographs, followed by the next innovation c. 2500 in Fara where we see the early attempts at encoding phonetics to writing to reduce the number of individual signs needed and the burden on agreeing their meeting through cultural convention, as well as the application to the new conqueror language of Akkadian. See [**Damerow/1999w**].

The earliest writing appears between 3500 and 3100 BCE depending on which of the proto-writing materials one is willing to admit. Regardless, there still between 500 and 1000 years before the first readable cuneiform dating to c.2500 BCE (the school texts of Fara). [**Nissen/1986**]

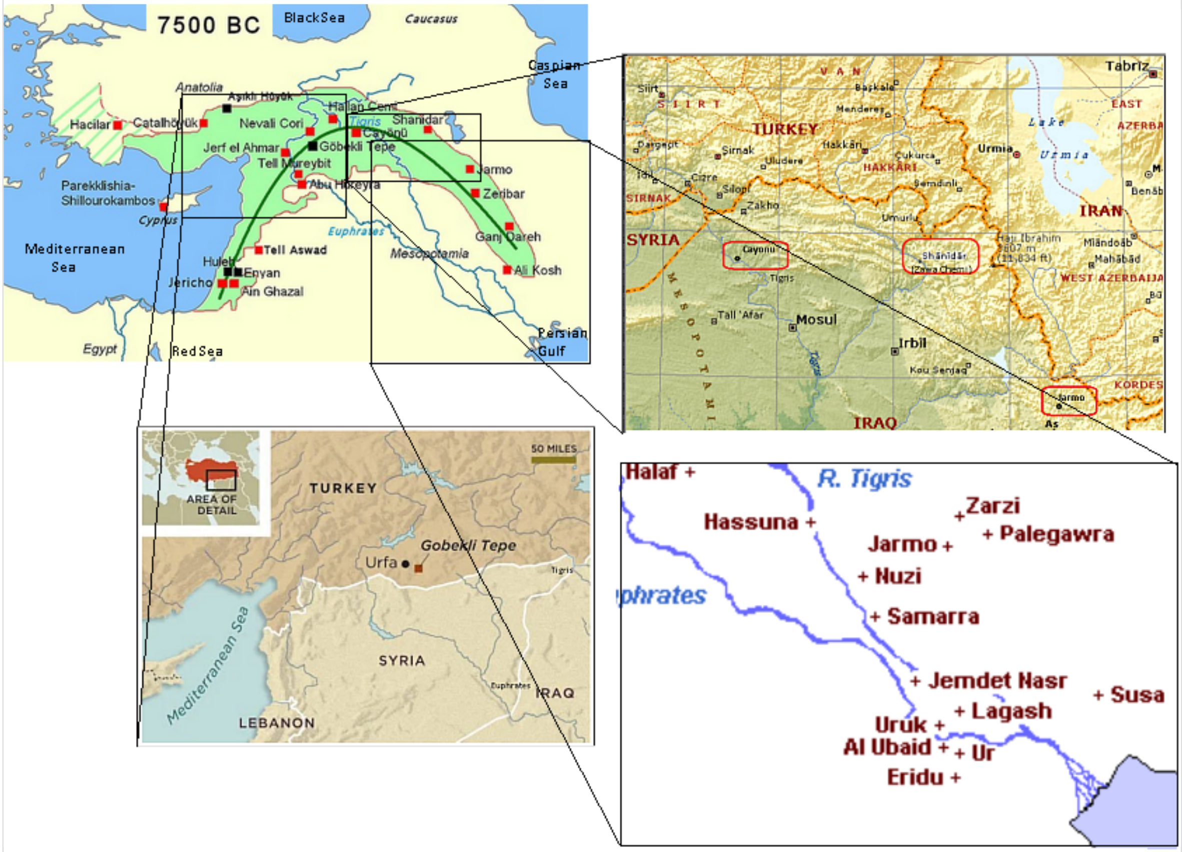
Adam Falkenstein (1936) published ATU1: The Archaic Texts of Uruk, which recorded the archaic signs occuring in the first 620 tablets found at Uruk in the first three seasons of excavations there. In the 1980s, Hans Nissen, a student of Falkenstein, launched the Berlin Project that aimed to publish all texts found since Falkenstein's publication. The difference was that since Falkenstein there was a group of texts the so-called "Lexical Lists" which went from 0.5% of known texts in 1936 to 15% in 1986 (50 years later) and were word-for-word ancestors of the 'schooltexts' from Fara (Shurrupak) and which are almost fully comprehensible. This has brought almost 70% of the archaic signs to be identified. (The remaining 85% of archaic texts are so-called economic or administrative texts, i.e. these are receipts, lists of expenses, of animals, of all kinds of goods, of raw materials.) [**Nissen/1986**]

Why writing? “It was the need to control an expanding economic unit (the Eanna temple) that prompted the introduction of controlling devices better suited for managing large quantities of information than the human memory.” [**Nissen/1986**, p.324] Writing was an advancement from other innovations in managing complex economy, namely cylinder seals, tokens, clay envelopes (bullae), numerical tablets, ideographic tags, and finally numero-ideographic tablets, and ultimately their standardization into cuneiform. [cf. **Hoyrup/1991**] “Writing appeared as the final solution to a number of economic problems which had probably been accumulating for a long time.” [**Nissen/1986**, p.326]

The evolution of writing over 1,100 years, from proto-cuneiform in Late Uruk period (3,100 BCE) to syllabic cuneiform during the Ur III period (2000 BCE). It took over 1,000 years to go from the first signs to the Ur III signs. (40 generations of 25 years each).

LU2 A Lexical List of Standard Professions, from 3200 BCE (Uruk IV) through to the Fara schooltexts.
Source: **Englund/1998**, p.104, Fig 32.
Transliteration: **ORACC**
Tablet attestation: **MS 2429** (from Umma, c.3200-3000 Uruk III period)

Appendix 3: Near Eastern Cultural History: from Shanidar to Uruk

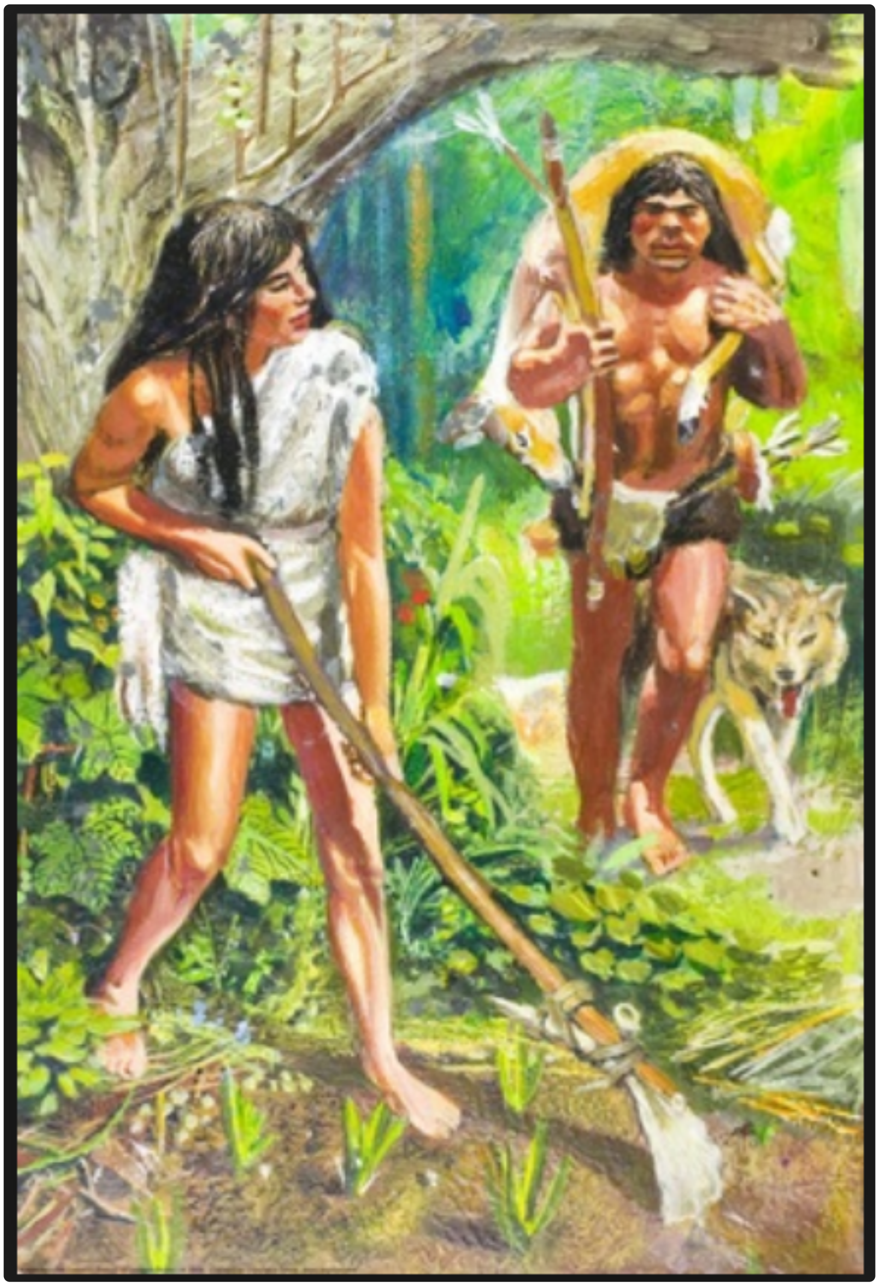


Paleolithic Sites (Shanidar mod. Zawa Chemi, Pelegawra) and Mesolithic Settlements c.7500 BCE including the earliest settlements in southeastern Anatolia (Gobekli Tepe) and N. Iraq (Cayonu, Jarmo)

- **Paleolithic** – from the first stone tool wielding hominids (3.3 mya) through the four ice ages to hunter-gatherer subsistence mode (Neanderthals and Homo Sapiens) e.g. in **Shanidar Cave (65,000 BCE)**, a Middle Paleolithic (Mousterian) site in the Zagros mountains of Iraq/Iran/Turkey border. The last stage is **Upper Paleolithic (50,000 BCE)**

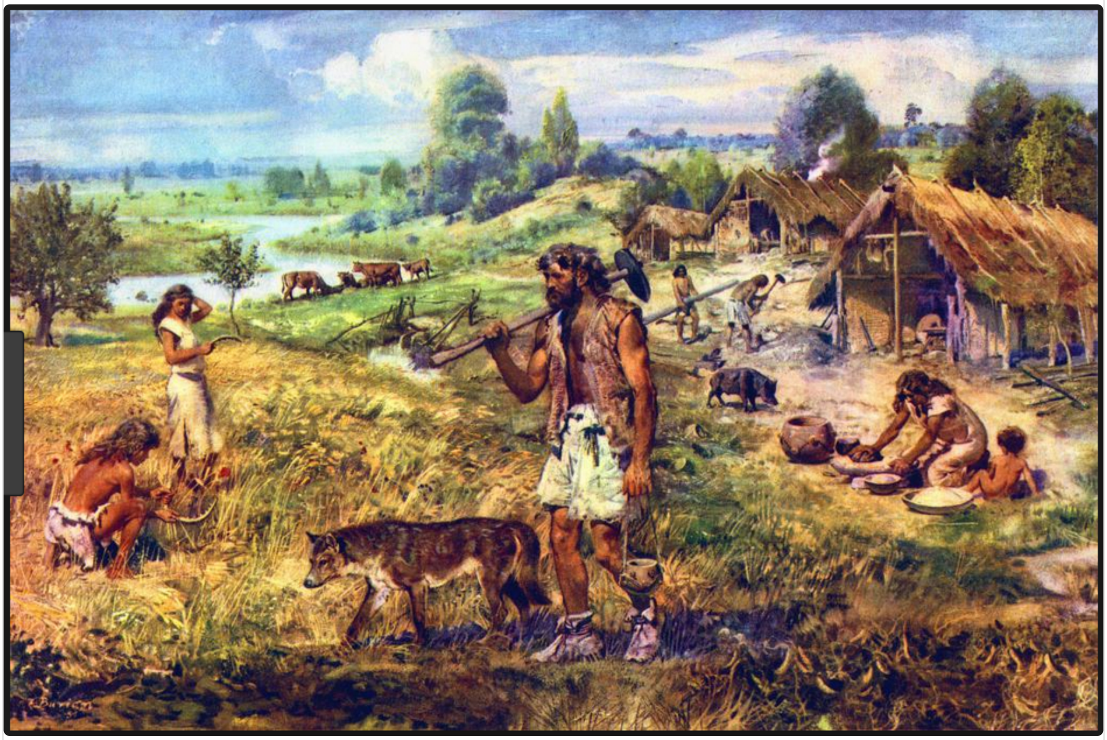
Artists conception of communal living in Shanidar cave at the end of the fourth glacial advance (ca. 10,000 BCE)

- **Mesolithic (18,000 BCE)** – after the last (fourth) ice age, transition between hunger-gatherer to increasingly sedentary subsistence mode. **Gobekli Tepe (9130 BCE)** on the Anatolian/Syrian border is the earliest known temple site and is unique in that there is no evidence of sedentary living associated with it, or cultivation of grains. The limestone pillars weigh approx. 10-50 metric tons and would have required atleast 500 adults to move and place. Starting to experiment with taming nature: wild cereal cultivation, domestication of the dog, domestication of other animals (sheep, goats), mix of semi-settled and nomadic herders (e.g. **Kebaran culture** in the Levant from 18,000 BCE, **Natufian culture** in the Levant from 12,500 BCE, first evidence of bread 14,500 BCE and beer 11,000 BCE)



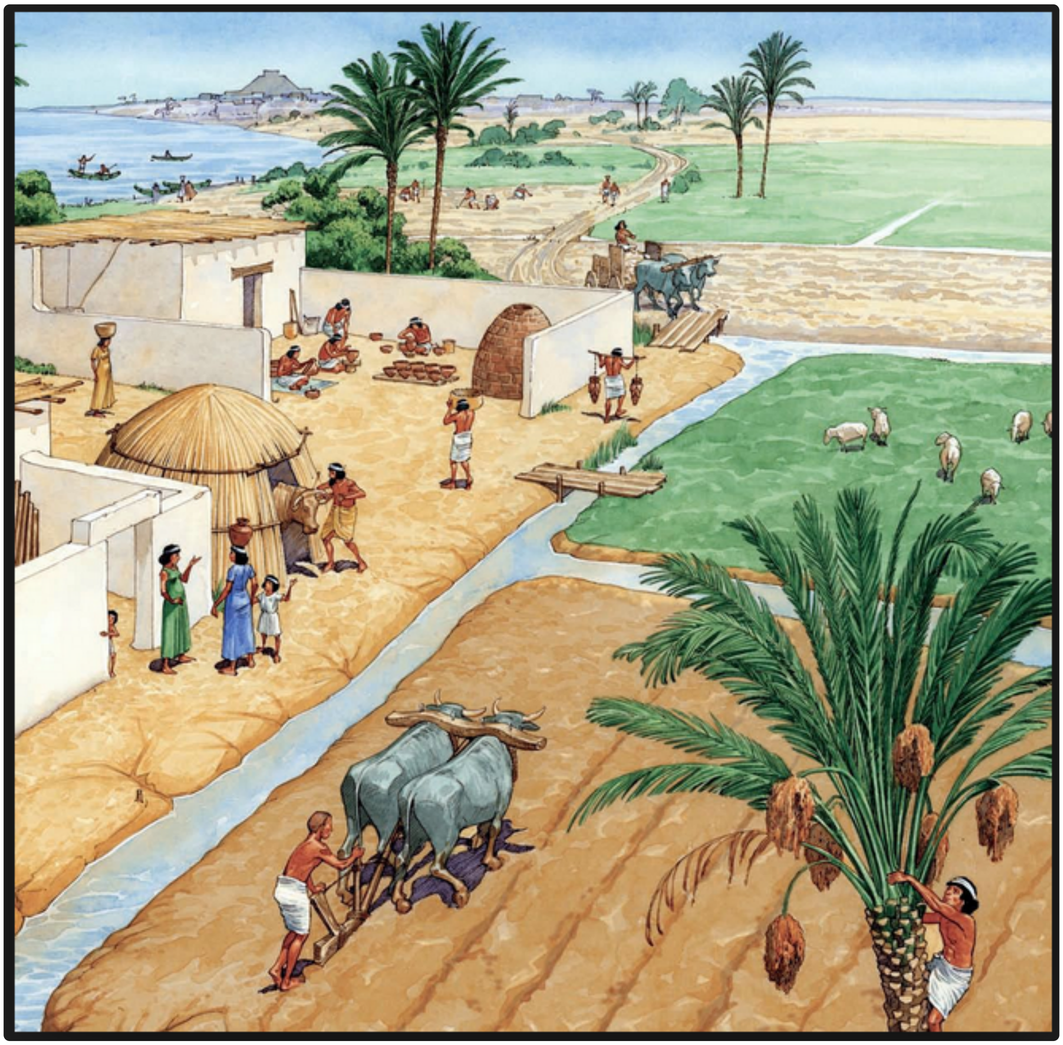
Artists conception of Mesolithic life, combination of hunter and gatherer lifestyle.

- **Neolithic (10,000 BCE)** – culture forming around settlements, herding, farming, but also transhumance seasonal migration between lowlands and highlands. Type site: **Jarmo (7090BCE)**, an agricultural community of 150 people, in the foothills of the Zagros mountains

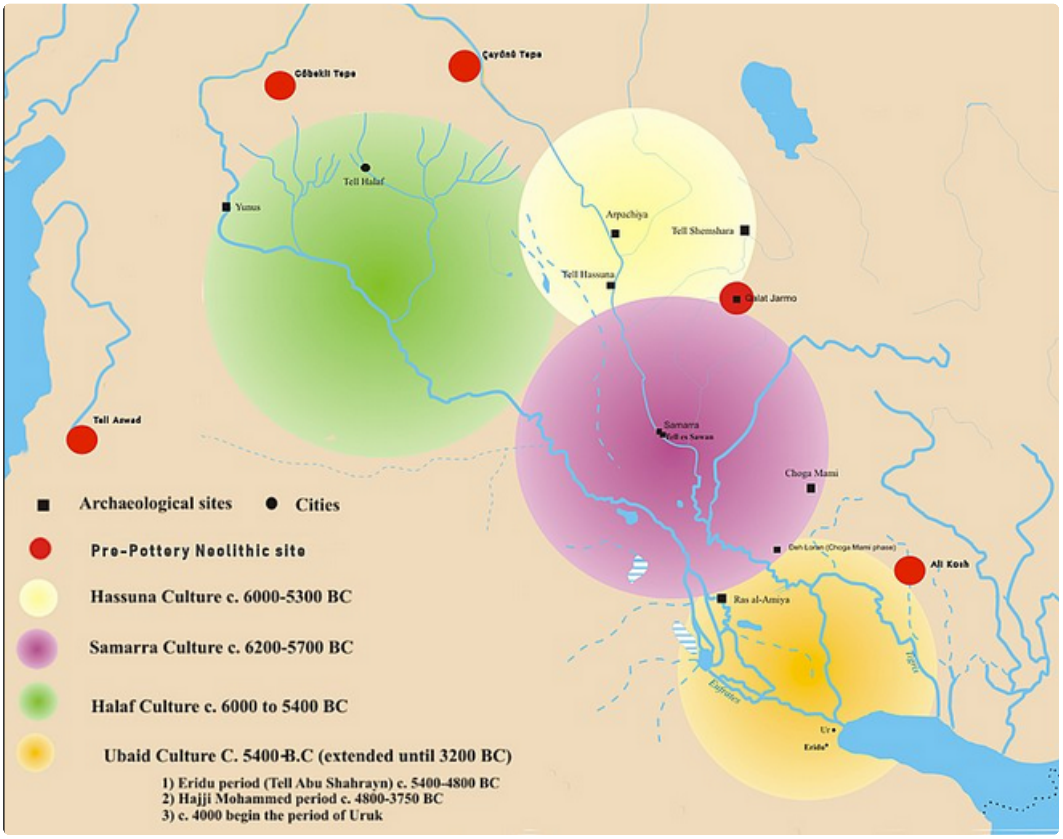


Artist conception of Neolithic lifestyle.

- **Ubaid (6500-3800 BCE)**/**Chalcolithic (4500 BCE)** – transition to permanent unwalled settlements with specialized craftspeople (potters, weavers, metalworkers), cultivation of grain under arid conditions through the use of irrigation canals (some up to 5km long) requiring large collective labor efforts, the growth of an extensive trade netowrk, and the building of temples. First known settlement in S.Mesopotamia is **Tell el-Ouelli (Ubaid 0)** (6500 BCE-5400 BCE), **4km SE of Larsa**, 25km SE of Uruk. Next is **Eridu (Ubaid 1)** up to 4,000 residents in 20-25 hectares, irrigation agriculture, limited use of copper metal tools, expansion of art and aesthetics, and the beginnings of stratification of society, professional specialization, and the clustering of villages around centers

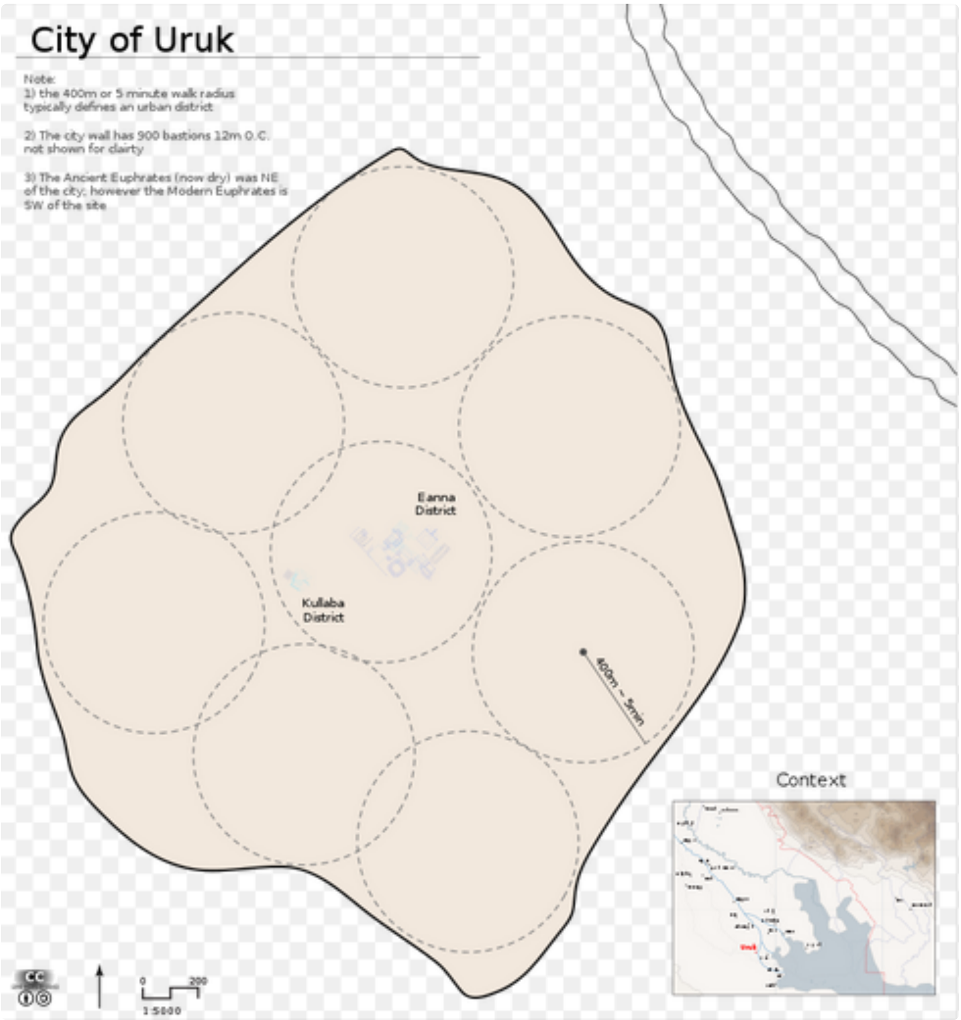


Artists conception of Ubaid life (unwalled settlements, communal labor, irrigation agriculture, copper supplementing stone and wood tools)



Ubaid period cultures, c.6000 BCE onwards.

- **City-State Period (4,000 BCE-2,900 BCE)** – early bronze age, expansion of settlement size to large cities with walls (Uruk, Ur, Susa), **with up to 50,000 residents in 6 km2 (Uruk c.2900BCE)**, hierarchical society with an established elite (temples and lords), warrior class, slavery, long distance trade, large surpluses and the controlled use of labor for prestige buildings – emergence of writing, the state, arithmetic, ancient book-keeping. **Uruk city** (founded in Eridu Ubaid 1 period 5,000 BCE onwards) originated as two separate temple sites to Innana and An (Kullaba district).*See **FAQ1** for discussion of **middle-chronology dating** of Mesopotamian events from Early Dynastic onward*



Uruk City, founded in Ubaid 1 (Eridu) period 5,000 BCE as two temple sites, Eanna (to Inanna) and An, consolidated into a single walled urban site, eventually with an intricate inter-city canal system allowing heavy goods (wood, stone, etc.) to be brought into the city by boat from outposts, colonies, and distant trading partners (Venice in the desert)

For history after ED period, see Part 2: The Mathmatics of Uruk and Susa, Appendix

Appendix 4: Ancient Stone Age (Paleolithic) and the Four Ice Ages

The Paleolithic epoch begins with the appearance of **the earliest stone flake tools 3.3mya near Lake Turkana in Kenya**.

Australopithecine disappeared from the fossil record about 1mya, replaced by Homo genera of the Hominidae family. But for a time there is overlap between co-existing distinct human species.

From 2.5 mya to 1.3mya we have Homo habilis (`handy man'), with more sophisticated stone tools near Oldowan in Tanzania.

From 1.9 mya to 400k ya we have Homo erectus.

Note that, during this period from 1.9mya to 1mya, for almost 900,000 years, there were at least 3 species of humans overlapping in the same range in eastern and north-eastern Africa (australopithecus, homo habilis, and homo erectus).

Three more species of humans fill out the story: Homo sapiens, **appearing 230kya (Omo 1 site in Ethiopia)** and even earlier at **315kya (Morocco site)**, Homo sapiens neanderthalis appearing about 130kya, and Homo sapiens sapiens (anatomically modern humans) appearing about 100kya.

Throughout this period, waves of human ancestral migration leave Africa and populate the earth. Homo erectus remains of 1mya are found in the Jordan valley (Ubaidya site).

By 700kya, Homo erectus man has learned to control fire, potentially accelerating the evolution of man to homo sapiens as a more protein rich diet could be consumed through cooking, leading to brain expansion. This theory is contested by the discovery of small brained human ancestors who were every bit as smart as the larger brained successors. (citation)

By 315kya, Homo sapiens had emerged based on **fossil discoveries at site in Morocco**
By 230kya, Homo sapiens had emerged, based on fossil discoveries amidst volcanic ash **discovered at the Omo 1 site** in the Ethiopian portion of the Rift Valley in East Africa.

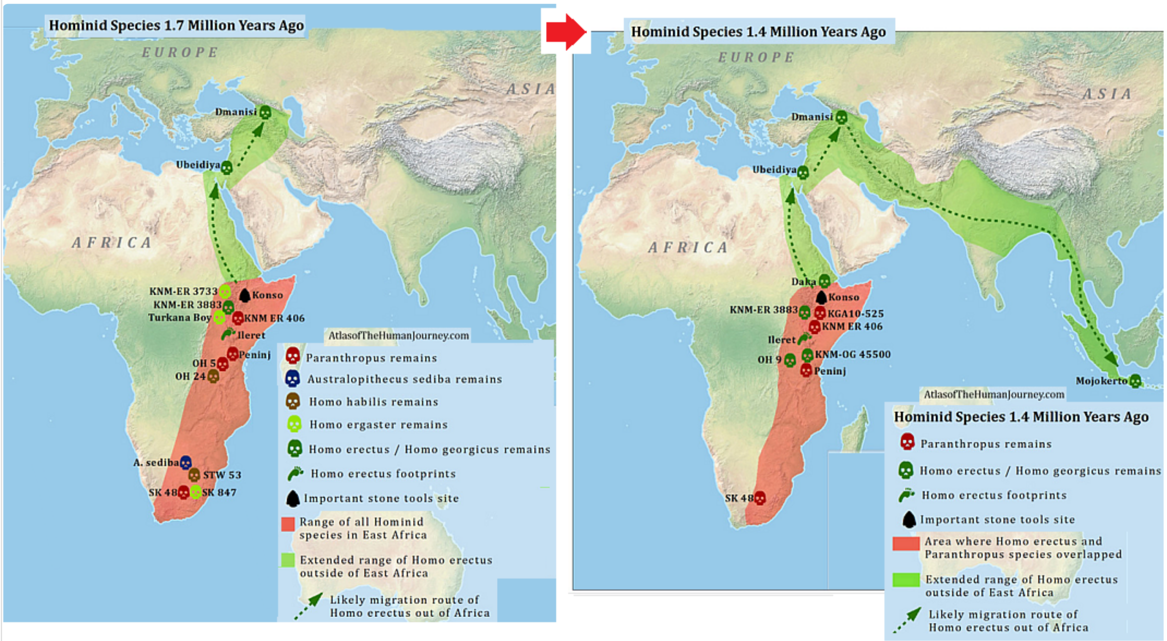
By 100kya, most of the Earth has been populated by hominid ancestors.

The last glacial period began 110kya and lasted to 11,700 years ago, or to 9700 BCE.

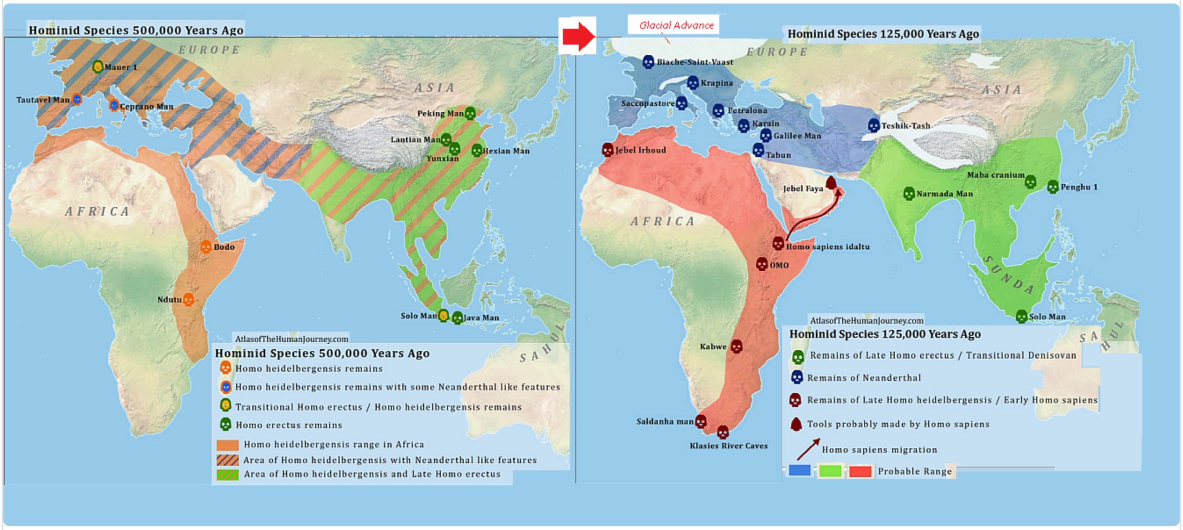
By about 50kya, a wave of anatomically modern humans (homo sapiens) left Africa and moved through the Fertile Crescent (**Map of Pre-historic sites**)

In this early period (48kya-35kya), we have both Neanderthal and Homo Sapiens occupying the same region, with the Neanderthal skeletons of Shanidar Cave in the Zagros mountains providing possibly the earliest evidence of human assault on Neanderthals. The Skeletons of Shanidar Cave, in the Zagros mountains of Kurdistan in northern Iraq, are Neanderthals from 65kya to 35kya. 10 nearly complete Neanderthal skeletons provide a remarkable picture. Neaderthals cared for their wounded and buried their loved ones in graves. There is evidence of murder of `Shanidar Three' by a low-mass, low-kinetic energy projectile weapon, either by fellow Neaderthals, or by projectile-carrying homo sapiens who had entered the region. In contrast with the killing projectile, Neanderthals used heavy huntings spears thrust with great force at close range into their prey (large mammals). Humans had mastered lighter projectile spears, throwing these deftly and with some accuracy from further away. (**Article 1**, **Wikipedia: Shanidar Cave**, **Article 2**, **Article 3**)

See Appendix 5 for the continuation of the story into the Middle Stone Age (Mesolithic) and later.



Hominid Migration out of Africa, 1.7mya to 1.4mya
Source: AtlasOfTheHumanJourney.com



Hominin Migration, 500kya to 125kya, source: AtlasOfTheHumanJourney.com

Appendix 5: Culture in the Near East: From Mesolithic (after the last ice age) to the Neolithic (rise of sedentism)

Middle Stone Age (Mesolithic): After the Last Ice Age but before Sedentism

References: Charvat/2002, Nissen/1988

By 14kya, the Paleolithic era is ending with the last ice age giving way to the Younger Dryas inter-glacial period (lasting to the present).

By this time 14mya man has mastered the bow and arrow, and we have evidence of domesticated dog living amongst men (**evidence at Palegawra site (1975) in the Zagros mountains**. How did this come about? Russian and US scientists have **identified a gene (SorCS1) in foxes that leads to exaggerated friendliness and tameness behaviour (2018)**. A tameness gene may have made a few wolves seek human affection. If female, this may have led to pups with the same gene, and over the course of hundreds of generations of animals, to dogs.

The change in climate and habitat triggers the start of the Mesolithic epoch in the transition from Paleolithic (nomadic hunter-gatherer) to Neolithic (settled farming).

As the last ice age receded around 12kya, the environment began to yield much more plentiful food sources for hunter gatherers.

In the Mediterranean and Near East, wild grasses and cereals (wild barley, einkhorn and emmer wheat) increased, accompanied by lentils and various pulses.

Hunting moved from indiscriminate killing of a wide range of animals to a focus on a few species, particularly wild sheep, wild goats, and onager (wild donkey), supplemented less intensely by deer, wild boar, wild cattle, hare, wolf, fox, various birds, and other small mammals.

Gathering was turtles, hedgehogs, snails, other molluscs, and plant food.

For tools, they processed stone, bone (awls, knife handles, etc.), wood. To form axes, they used bitumin, a naturally occurring sticky tar, to attach stone or obsidian (black volcanic glass) blades into notched handles.

Stone tools were typically made of chert or flint, obsidian or quartz (**hardest material**)

> Abundance spurred curiousity, new resources were opened (a variety of stones, raw copper, bitumen) and new technologies such as grinding and polishing stones and even the first steps in chemical production (lime plasters), were introduced.” (**Charvat 2002**:10)

Even in this period of plenty, there is evidence of violence between human beings.

By 11kya, they started to settle down in semi-fixed homes and experiment with mixed mode living — hunting, gathering, herding, and with wild crops.

By 10kya, man had domesticated other animals, not unsurprisingly first sheep and goats, given the heightened contact through hunting, and then cattle. By 9,500 ya, evidence exists of domesticated pigs.

By this point, housing was kept scrupulously clean, with lime plaster or clay interior walls, lowered floors, spiritual or ritual objects – statuettes, grave goods, body ornaments (necklaces, bracelets, rings). (**Charvat 2002**: 13)

> “The essential characteristics of all human communities up to recent time—economic specialization, social differentiation and complex spiritual reflection of the visible world—may be documented in this period of time.” (**Charvat/2002**:15)

New Stone Age (Neolithic) and Rise of Sedentism

Now we enter the Neolithic period, in the Near East this is from 9500 BCE onwards.

In agriculture there is experimentation with emmer wheat and einkhorn wheat, also cultivated peas, lentils, six-row bread wheat, oats, rye, linseed, and flax. They gathered wild cereals and also pistachio nuts from the highland woods.

Some of the dwellings (e.g. at Umm Dabahiyah) now have “fresco paintings geometrical patterns and figural scenes (an onager frieze, a hunting scene)” (Charvat 2002:19) — fresco paintings are with colored powders applied to fresh plaster so that upon drying, the painting is an integral part of the wall.

There is pottery, and ornamental decorations on pottery, houses with complex structures, including stairways for roof access, kilns for firing pottery, textiles and woven baskets.

In agriculture there is artificial irrigation leading to larger crop yields (sites Choga Mami and Tell es-Sawwan inter alia). There is also clear evidence that Neolithic peoples were experimenters, cross-breeding cereal grasses to obtain domesticated variations that are in use to this day (four-row emmer wheat, six-row variations, with non-shattering stems in order to preserve the grains when harvesting). Charvat 2002:30

This was their main achievement – the advance and experimentation in securing additional food sources and improving and perfecting these through genetic interventions.

Dogs were used to assist in both shepherding and hunting. As shepherding added more species (goats, sheep, cattle, pigs), hunting targets changed away from wild goats to gazelle and onagers, presumably because wild goats would be added to the herd. Gazelles and onagers are harder to hunt, requiring the coordination of numerous hunters with a single purpose. They also consumed more fish, mussels, turtles, and crabs.

The settlements consisted of several houses, in some cases (Tell es-Sawwan) with a fortification ditch (3m deep, 2.5m wide) surrounding the houses, together with a rampart with buttresses (reinforced defensive walls). Houses were built of clay bricks, sometimes formed in molds, and the floors bore an occasional coating of bitumen (tar) or gypsum, otherwise reed mats, or stamped earth. Some of the village streets were paved.

By Neolithic time, civilization was complete — societies had structure, religion, economic specialization, surplus food, art, and community.

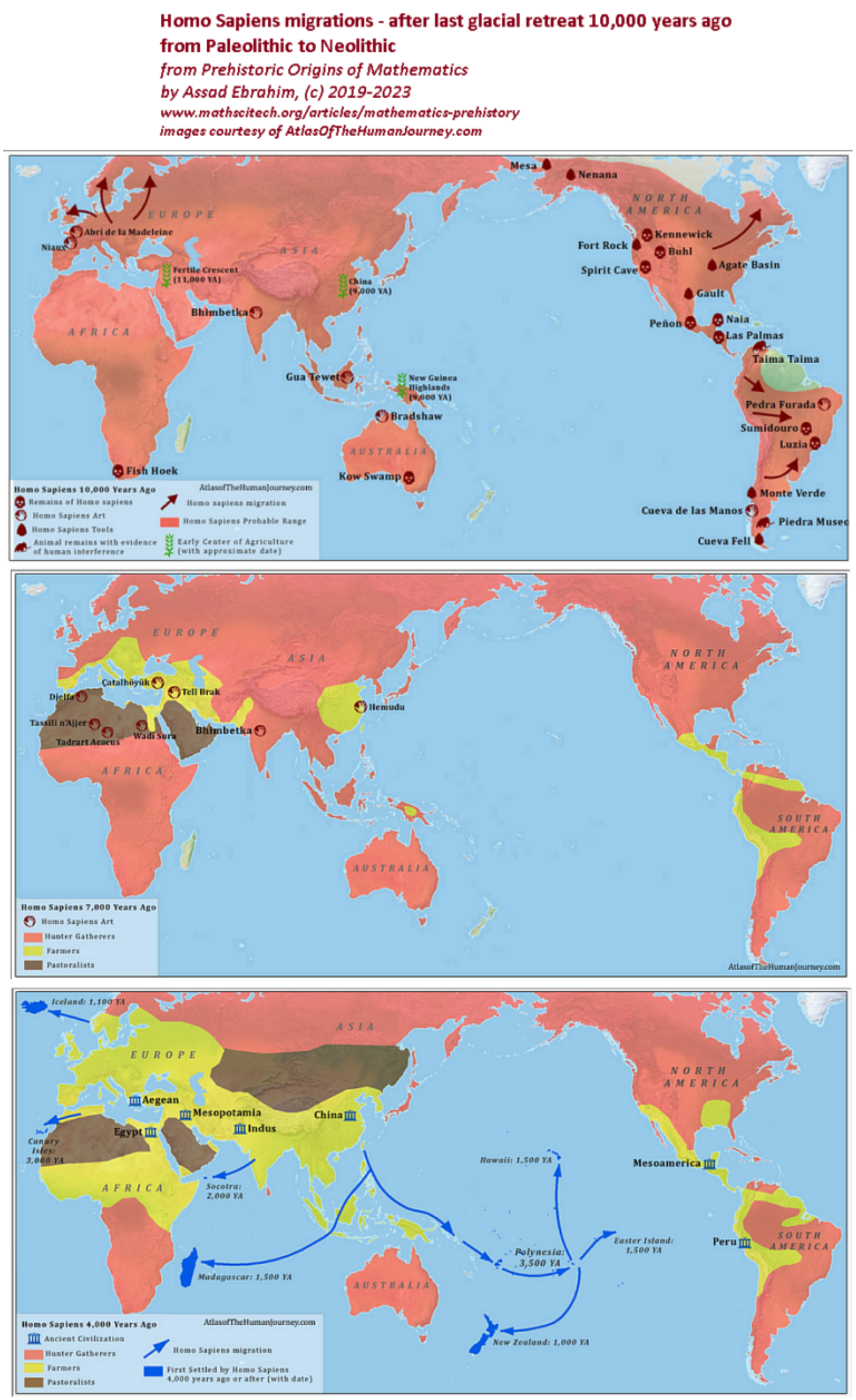
Planting and harvesting of grain was in place by 6900-6000 BCE.

> “The Neolithic food-producing economy was no fun … human remains at the site show evidence of physically demanding work, including collapse of neck vertebrae due to carrying heavy loads on the head (remember that the wagon would not be invented until the Chalcolithic period [from 4000 BCE on].” The digging, threshing, grinding, harvesting, irrigation, water carrying, load carrying — all of this was human labor. (Charvat 2002:32)

One wonders if in the Neolithic times ideas of slavery led to raids on nearby settlements in order to coerce others to carry out food-producing work.

Lifecycle of lambs and sheep: “Lambs are born around Christmas (for confirmation by the Near Eastern data see Wright, Miller and Redding 1980, 271; Wright, Redding and Pollock 1989, 108–109; Hruška 1995, esp. pp. 82–83) and in May they are usually grown enough to walk even over heavy ground and to be weaned so that sheep can be milked from that time on. In May the shepherds with their herds usually ascend the summer pastures whereupon the sheep are sheared and new wool employed to settle all accounts, debts and obligations that the shepherds or their masters might have incurred before, the season of cheesemaking following in the months of June and July. (Charvat 2002:39)

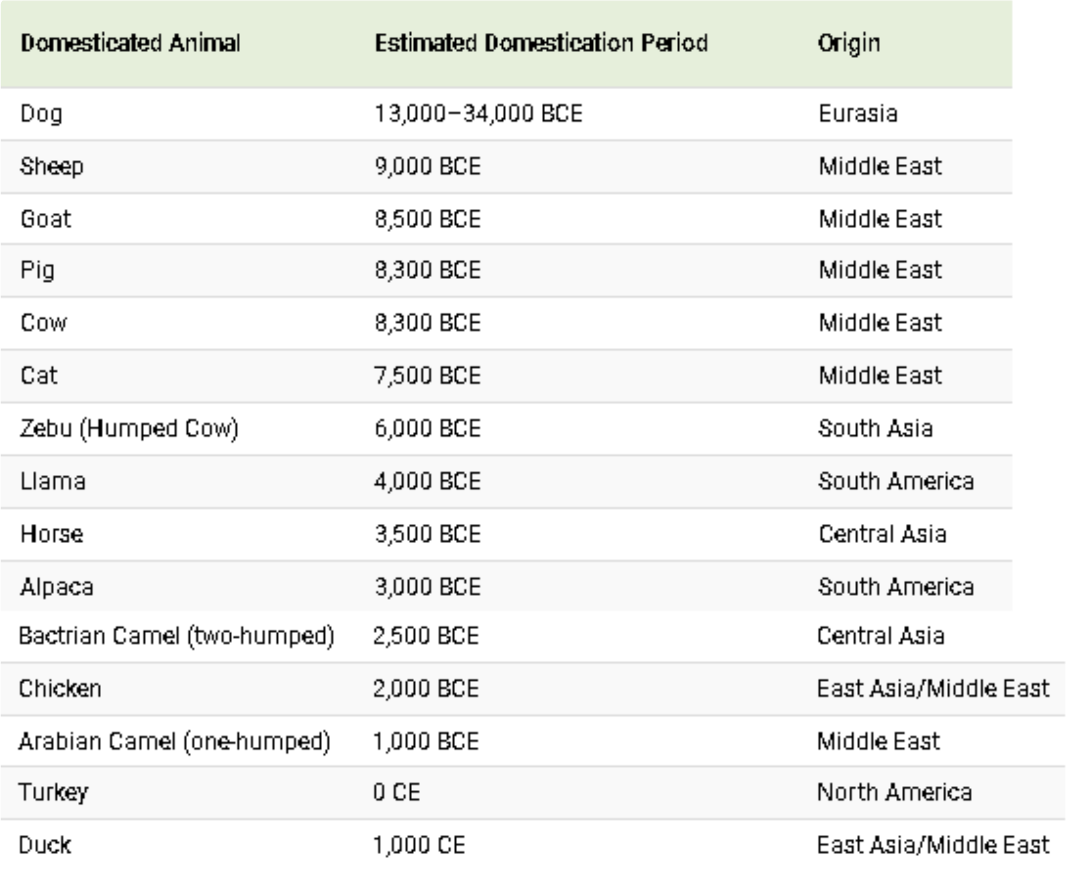
There is some evidence that Neolithic cultures were migratory, moving seasonally between lowlands (winter) and highlands (summer), and taking advantage of whatever combination of subsistence methods worked in each circumstance. So there was agriculture, herding, hunting, gathering, but looks like there may have been migrations twice a year, (Charvat 2002:47) I.e. the sites were permanent but the people in them were not (Charvat 2002:40) Non-nomadic domesticates are the pig, which cannot travel long distance. Another sign is larger cemeteries indicating territorialization of human communities. (Charvat 2002:39)



Homo Sapiens migrations - after last glacial retreat 10,000 years ago
from Paleolithic to Neolithic
from Prehistoric Origins of Mathematics
by Assad Ebrahim, (c) 2019-2023
www.mathscitech.org/articles/mathematics-prehistory
images courtesy of AtlasOfTheHumanJourney.com

Homo Sapiens migration – from Paleolithic to Neolithic, 10,000 years onward
source: AtlasOfTheHumanJourney.com

Appendix 6: Domestication of Animals



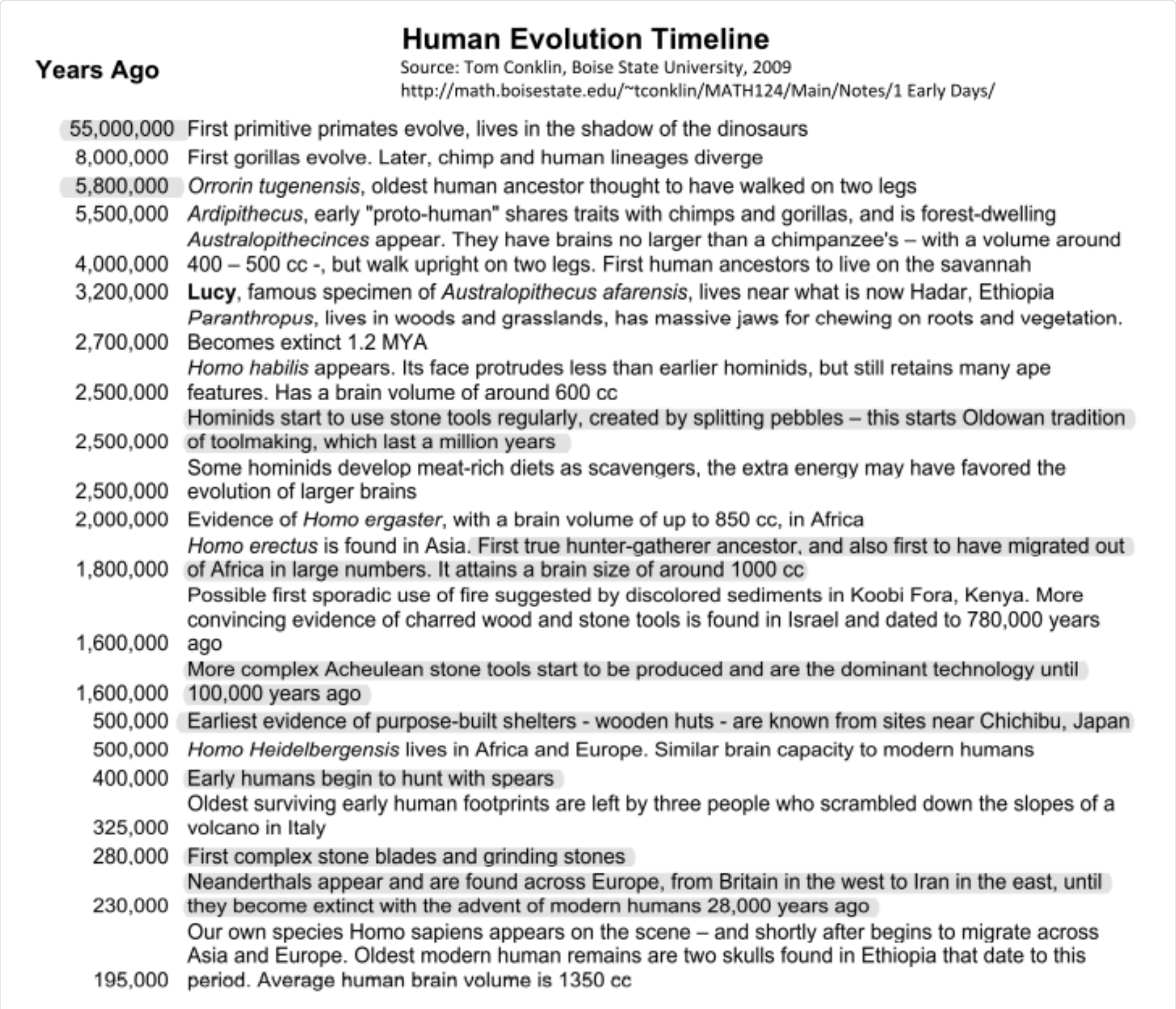
Domesticated Animal	Estimated Domestication Period	Origin
Dog	13,000–34,000 BCE	Eurasia
Sheep	9,000 BCE	Middle East
Goat	8,500 BCE	Middle East
Pig	8,300 BCE	Middle East
Cow	8,300 BCE	Middle East
Cat	7,500 BCE	Middle East
Zebu (Humped Cow)	6,000 BCE	South Asia
Llama	4,000 BCE	South America
Horse	3,500 BCE	Central Asia
Alpaca	3,000 BCE	South America
Bactrian Camel (two-humped)	2,500 BCE	Central Asia
Chicken	2,000 BCE	East Asia/Middle East
Arabian Camel (one-humped)	1,000 BCE	Middle East
Turkey	0 CE	North America
Duck	1,000 CE	East Asia/Middle East

Timeline: The Domestication of Animals. (**Source**)

Others:

- Donkey – **domesticated 5000 BCE in East Africa**. Entered mid-East c. 2500 BCE

Appendix 7: Evolution of Humanity

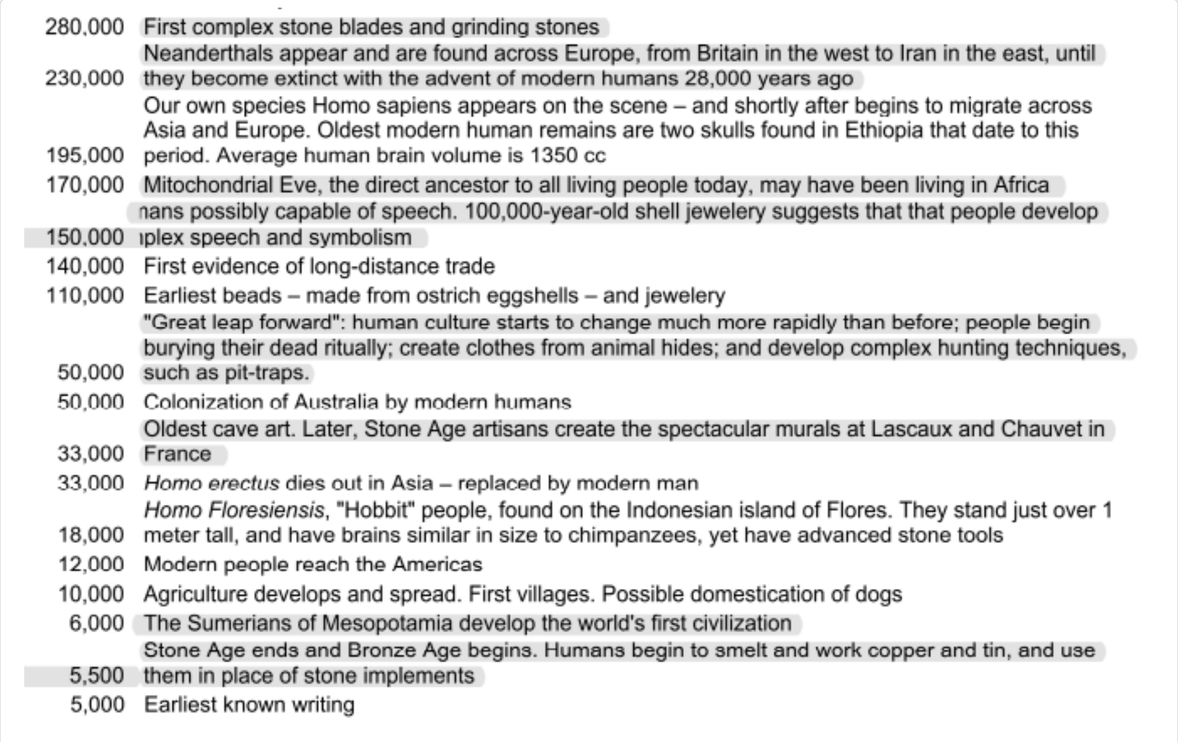


Human Evolution Timeline

Source: Tom Conklin, Boise State University, 2009
http://math.boisestate.edu/~tconklin/MATH124/Main/Notes/1 Early Days/

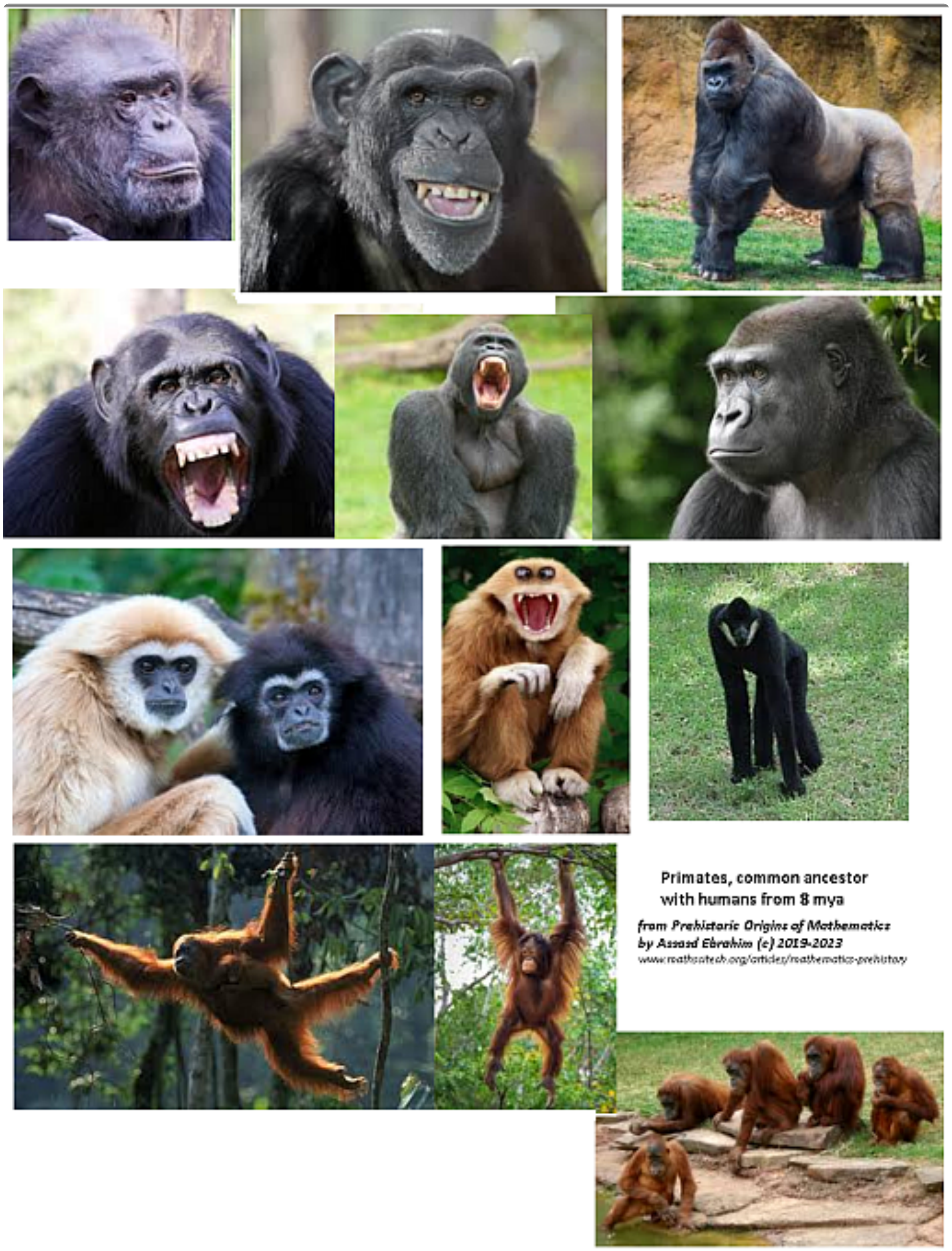
Years Ago	
55,000,000	First primitive primates evolve, lives in the shadow of the dinosaurs
8,000,000	First gorillas evolve. Later, chimp and human lineages diverge
5,800,000	*Orrorin tugenensis*, oldest human ancestor thought to have walked on two legs
5,500,000	*Ardipithecus*, early "proto-human" shares traits with chimps and gorillas, and is forest-dwelling
4,000,000	*Australopithecinces* appear. They have brains no larger than a chimpanzee's – with a volume around 400 – 500 cc -, but walk upright on two legs. First human ancestors to live on the savannah
3,200,000	**Lucy**, famous specimen of *Australopithecus afarensis*, lives near what is now Hadar, Ethiopia
2,700,000	*Paranthropus*, lives in woods and grasslands, has massive jaws for chewing on roots and vegetation. Becomes extinct 1.2 MYA
2,500,000	*Homo habilis* appears. Its face protrudes less than earlier hominids, but still retains many ape features. Has a brain volume of around 600 cc
2,500,000	Hominids start to use stone tools regularly, created by splitting pebbles – this starts Oldowan tradition of toolmaking, which last a million years
2,500,000	Some hominids develop meat-rich diets as scavengers, the extra energy may have favored the evolution of larger brains
2,000,000	Evidence of *Homo ergaster*, with a brain volume of up to 850 cc, in Africa
1,800,000	*Homo erectus* is found in Asia. First true hunter-gatherer ancestor, and also first to have migrated out of Africa in large numbers. It attains a brain size of around 1000 cc
1,600,000	Possible first sporadic use of fire suggested by discolored sediments in Koobi Fora, Kenya. More convincing evidence of charred wood and stone tools is found in Israel and dated to 780,000 years ago
1,600,000	More complex Acheulean stone tools start to be produced and are the dominant technology until 100,000 years ago
500,000	Earliest evidence of purpose-built shelters - wooden huts - are known from sites near Chichibu, Japan
500,000	*Homo Heidelbergensis* lives in Africa and Europe. Similar brain capacity to modern humans
400,000	Early humans begin to hunt with spears
325,000	Oldest surviving early human footprints are left by three people who scrambled down the slopes of a volcano in Italy
280,000	First complex stone blades and grinding stones
230,000	Neanderthals appear and are found across Europe, from Britain in the west to Iran in the east, until they become extinct with the advent of modern humans 28,000 years ago
195,000	Our own species Homo sapiens appears on the scene – and shortly after begins to migrate across Asia and Europe. Oldest modern human remains are two skulls found in Ethiopia that date to this period. Average human brain volume is 1350 cc

Human Evolution Timeline, from Primates 55mya to Anatomically Modern Humans 230kya

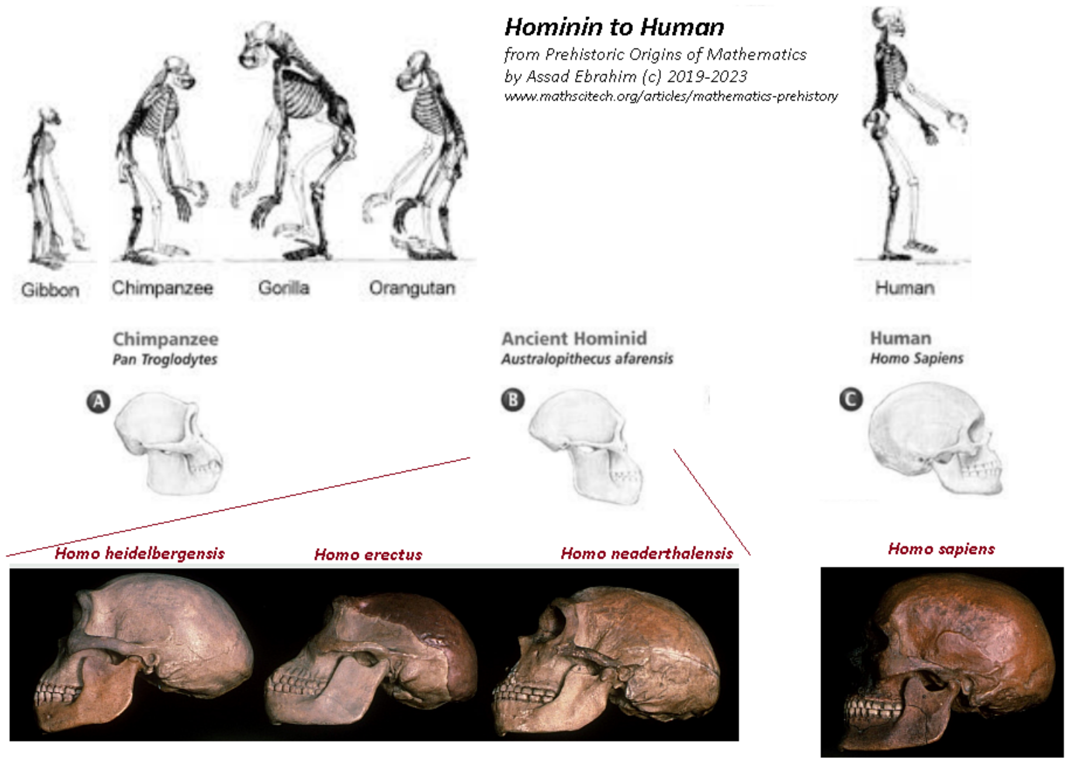


280,000	First complex stone blades and grinding stones
230,000	Neanderthals appear and are found across Europe, from Britain in the west to Iran in the east, until they become extinct with the advent of modern humans 28,000 years ago
195,000	Our own species Homo sapiens appears on the scene – and shortly after begins to migrate across Asia and Europe. Oldest modern human remains are two skulls found in Ethiopia that date to this period. Average human brain volume is 1350 cc
170,000	Mitochondrial Eve, the direct ancestor to all living people today, may have been living in Africa
150,000	nans possibly capable of speech. 100,000-year-old shell jewelery suggests that that people develop plex speech and symbolism
140,000	First evidence of long-distance trade
110,000	Earliest beads – made from ostrich eggshells – and jewelery
50,000	"Great leap forward": human culture starts to change much more rapidly than before; people begin burying their dead ritually; create clothes from animal hides; and develop complex hunting techniques, such as pit-traps.
50,000	Colonization of Australia by modern humans
33,000	Oldest cave art. Later, Stone Age artisans create the spectacular murals at Lascaux and Chauvet in France
33,000	*Homo erectus* dies out in Asia – replaced by modern man
18,000	*Homo Floresiensis*, "Hobbit" people, found on the Indonesian island of Flores. They stand just over 1 meter tall, and have brains similar in size to chimpanzees, yet have advanced stone tools
12,000	Modern people reach the Americas
10,000	Agriculture develops and spread. First villages. Possible domestication of dogs
6,000	The Sumerians of Mesopotamia develop the world's first civilization
5,500	Stone Age ends and Bronze Age begins. Humans begin to smelt and work copper and tin, and use them in place of stone implements
5,000	Earliest known writing

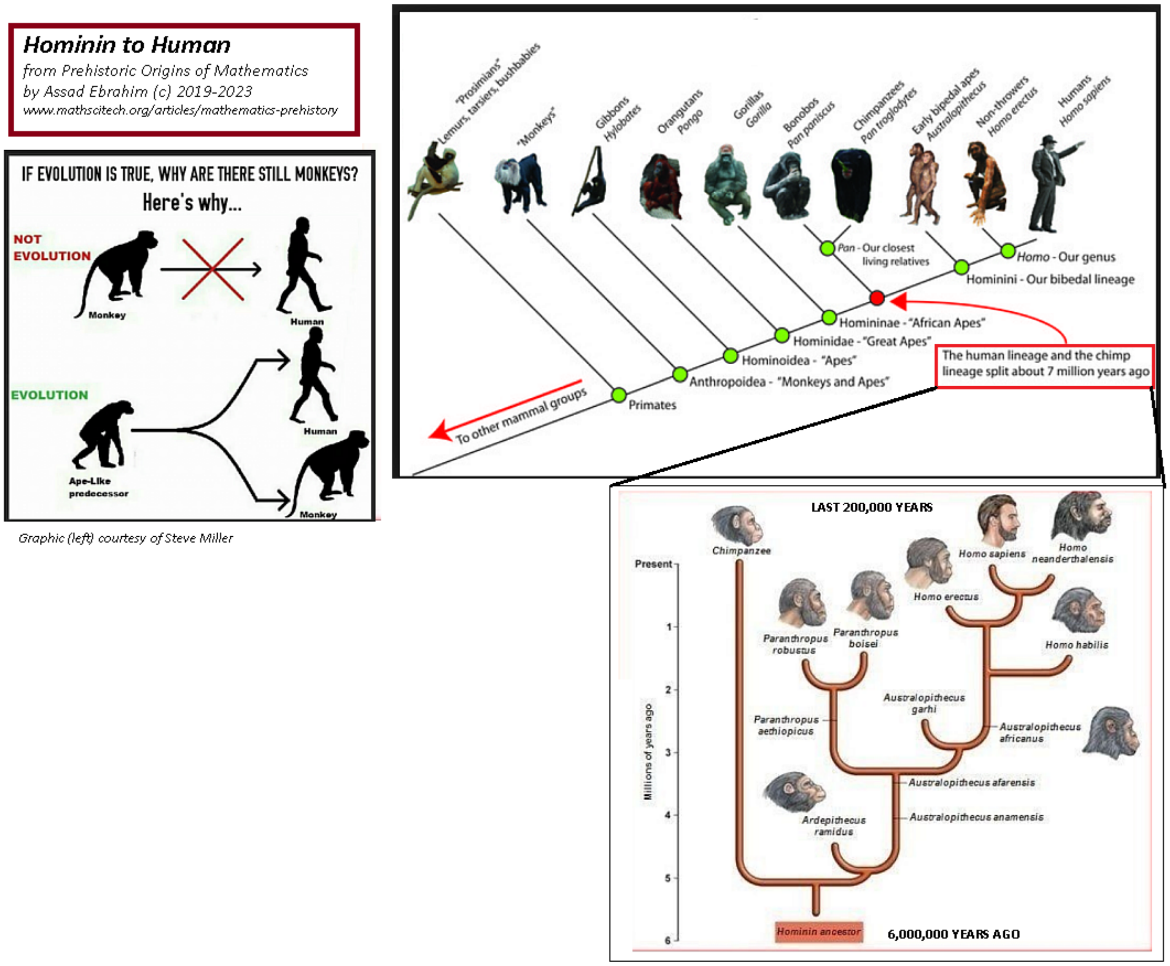
Human Evolution Timeline, from Neanderthals 300kya to the emergence of writing 5kya (c.3000 BCE)



Primates share common ancestor with humans, from 8mya

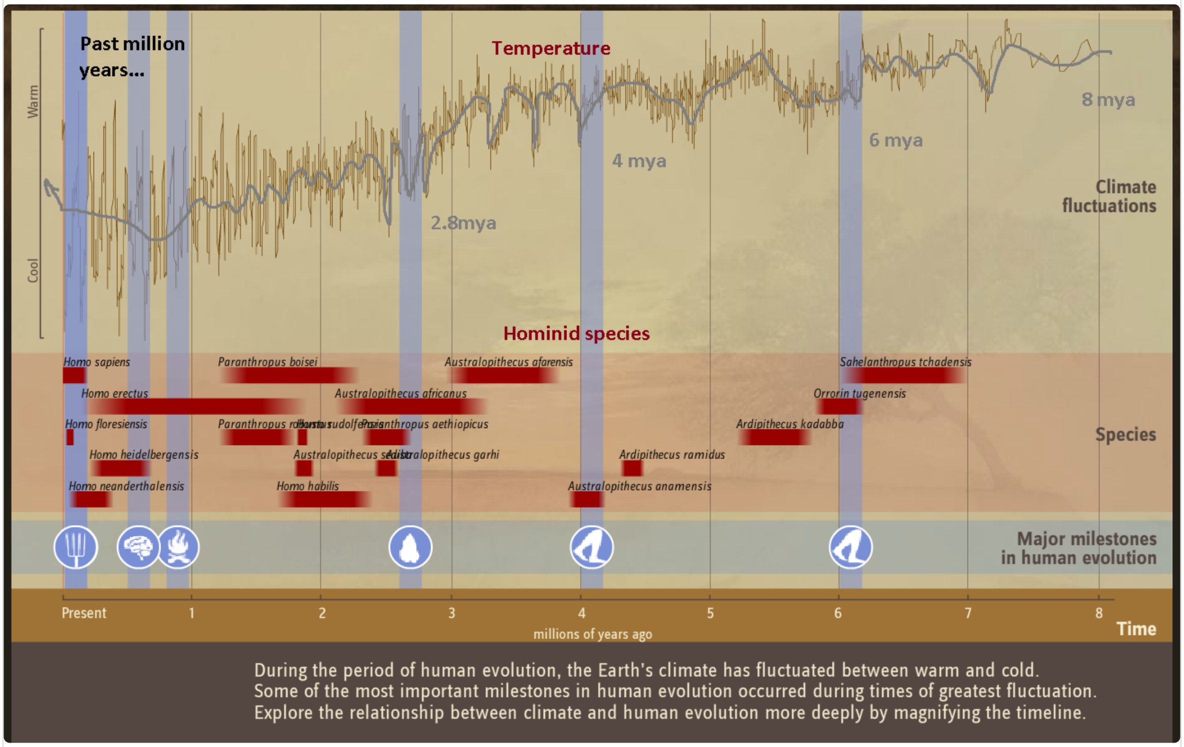


Hominin to Human (Part 1): Skeletal and Cranial comparisons



Graphic (left) courtesy of Steve Miller

Hominin to Human (Part 2): Last Common Ancestor (LCA) concept, and Family Tree of Hominins



Correlation of major climate fluctuation with emergence of human species

Appendix 8: Birth of the Universe to the Origins of Man

The Universe was formed 13.8 billion years ago (bya), +/- 21 million years (a measurement error of 15 years per million). **Within the first second**, the universe underwent a massive expansion accompanied by hypercooling and the formation of the fundamental forces and particles. **Over the next 20 minutes**, light formed (photons) and matter was created (mostly hydrogen and helium). The universe was a super-hot glowing fog (10^9 Kelvin K, or 10 billion degrees Celsius). It would take 370,000 years for the fog to dissipate, the universe to become transparent, and to cool a million-fold to 3000 K (2730* C). **Temperatures of 300K, suitable for liquid water and life** (the habitable era of the universe) were reached 10 million years after the big bang.

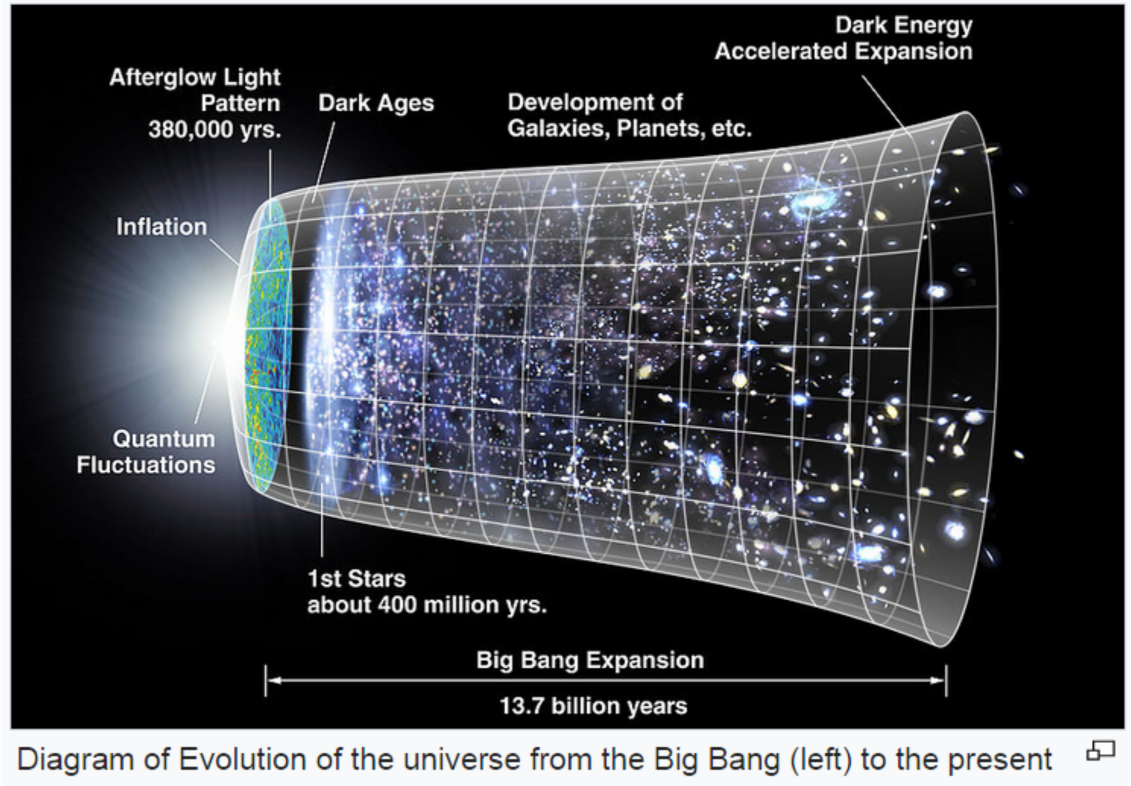


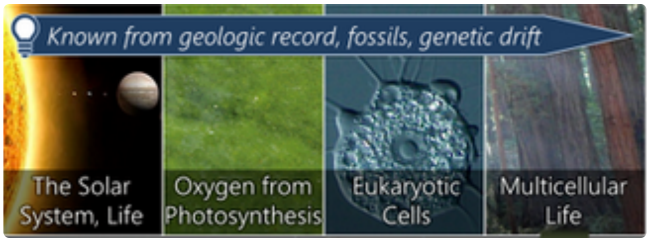
Diagram of Evolution of the universe from the Big Bang (left) to the present

Birth of Universe 13.8 billion years ago (bya). Source: Wikipedia: **Timeline of epochs in cosmology**

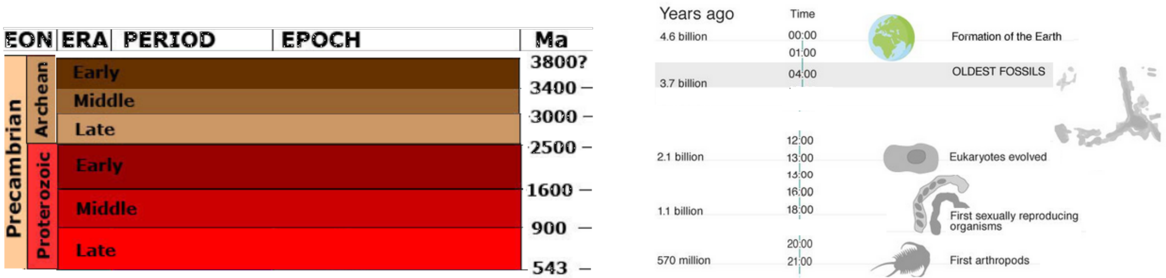
Star formation began after 100 million years, ending the so-called **"Cosmic Dark Ages"**. Stars began nuclear fusion reactions from 300 million years to form the heavier elements. The first galaxies appeared at 400 million years. Our Milky Way galaxy began to form after 700 million years, and would take the next 4.5 billion years to evolve until it acquired its spiral arms through galaxy collision (8.7 billion years ago bya). It would be another 4 billion years till our solar system formed.

The First 9 Billion Years. From the Birth of the Universe (13.8 billion years ago) to the formation of the Milky Way (760 million years after birth) to the start of the Milky Way spiral (5 billion years), to the formation of our solar system (9.2 billion years, 4.6 billion years ago) Source: Wikipedia **Matter Era of Cosmology**

Our solar system, including the Sun, the planets, and the Earth-Moon system, formed over a **100 million year period** between 4.6 and 4.5 billion years ago (bya). **What happened next on Earth** is still controversial, including by when the **molten earth cooled**, formed a crust, and contained surface liquid water. Also controversial is whether and to what extent a **Late Heavy Bombardment period** occurred during which large numbers of asteroids from the solar system's outer belt were pulled into the inner solar system and collided with many of its bodies, including the moon, and Earth itself. The **first evidence of life on Earth** follows immediately after this, with single-celled organisms (bacteria) first appearing in the fossil record 3.8-3.7 bya (Early Archaen Era). Over the next 3 billion years life evolved slowly with the **first exchanges of genetic material amongst prokaryotes occuring 2bya**, **first multi-celled organisms** appearing 2.7 – 1.6 bya (algae, amoebas, mold, fungus), and first sexually reproducing simple multi-cellular organisms appearing 1.1 bya (Middle Proterozoic era).

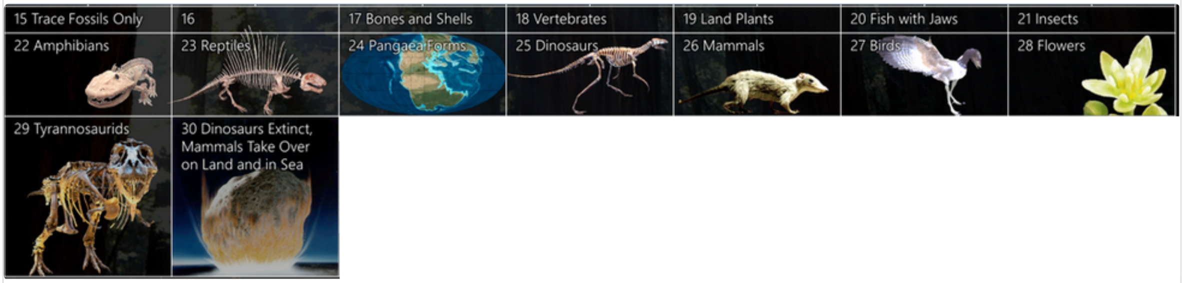


First 2 billion years on earth: from Earth formation (4.5 bya) to multi-celled organisms (2.6 bya)

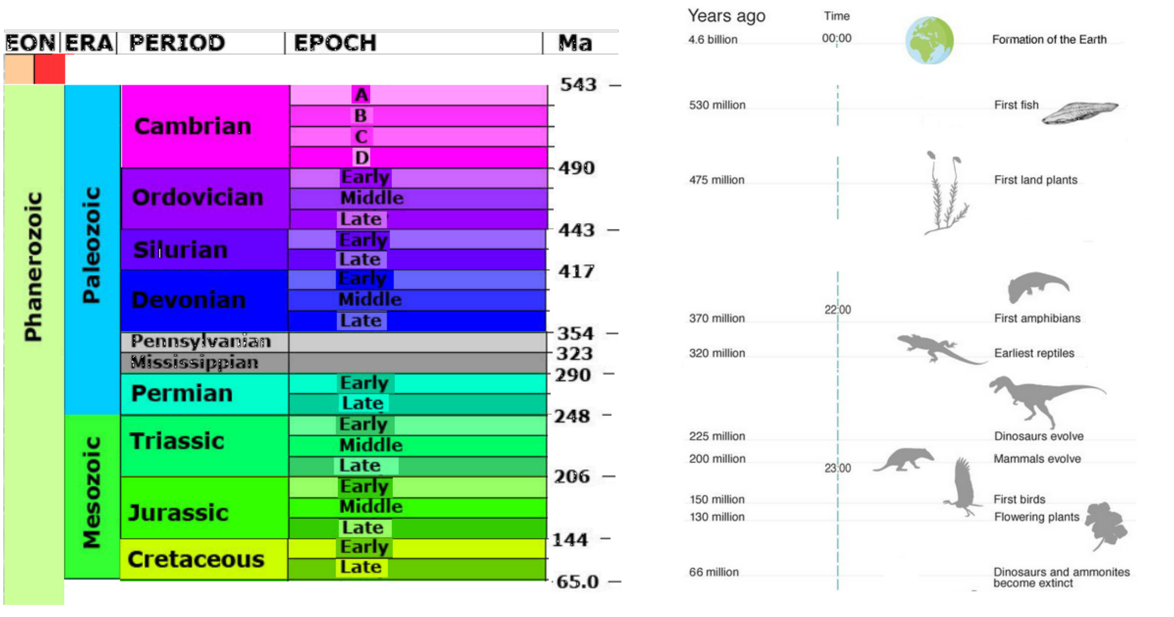


The first 4 billion years on Earth (4.6 bya): evidence of earliest life found 3.8bya, reproducing multi-celled organisms by 1.1bya, **the first animals (sponges) around 670mya**, and the **first arthropods (invertebrates)** by 570-555mya. Sources: **GeologyCafe.com** and Unknown

The next 500 million year period is fascinating, from the Cambrian explosion (500 mya) when life diversity accelerates rapidly to include fish, plants, reptiles, dinausaurs, birds, **mammals**, through to the cataclysmic event that ended the dinosaurs, triggered that cooling of the tropical earth and the rise of modern habitat and dominance of mammals. The cause of the acceleration in diversity is controversial. A **new paper (2018)** presents a provocative thesis – that the Cambrian explosion may have been triggered by the insertion of extra-terrestrial DNA into the earthly mix, brought in by a meteor or comet. Researches from the past decade have suggested that another major evolutionary event occured in the middle Triassic period, during the so-called **Carnian period around 252mya**. At this time, volcanic explosions are thought to have increased CO2 atmospheric content, warmed the global atmosphere, leading to 1-2 million years of rainfall on what had been previously an arid, dry Earth, and again accelerating diversity (dinosaurs, origin of mammals and many other modern forms of life.)



The Last 500m Years: From the Cambrian explosion to the end of the dinosaurs



Next 500 years, from the Cambrian Explosion (500mya) to the extinction of the dinosaurs (66mya)

In the aftermath of the dinosaurs about 66mya, **mammals flourished** and primates appeared (13 mya). By 3.8-3.5mya the earliest bipedal hominids (Austraopithecine) appear in Africa (Tanzania, Kenya, and Ethiopia), and the **stone age (paleolithic period)** begins (see **Appendix 4**).

Last 20m Years: from the appearance of primates (13mya) to the migration of anatomically modern humans out of Africa (c.60,000 years ago)

Download article (PDF)

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*transition happened c.3200 BCE in Uruk (aka Warka) in southern Mesopotamia. The rest of her many claims in subsequent publications have been demolished, in particular the claim that clay tokens were an accounting system in wide use across the Near East. See critical reviews by **Zimansky/1993**, **Englund/1993**, **Englund/1998**, and the use of contextual archaeology to close the case on Besserat's speculations, see masters thesis **Niemi/2016**, and **Bennison/2018***

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